Use of control charts in the production of concrete

by

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Amendments

Date	Amendments
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Symbols

AOQ	Average outgoing quality
AOQL	Average outgoing quality limit
f_{ci}	Individual test result for compressive strength of concrete
f_{cm}	Mean compressive strength of concrete
σ	Estimate for the standard deviation of a population
k	Statistical constant
f_{ck}	Specified characteristic compressive strength
L_l	Lower limit
n	Number of samples
q_n	Statistical constant that depends upon *n* and the selected AOQL
s	Sample standard deviation
UCL	Upper Control Limit
UWL	Upper Warning Limit
LWL	Lower Warning Limit
LCL	Lower Control Limit
C_{mra}	Constant giving the cement content increase required to produce a $1N/mm^2$ increase in strength
DI	Decision Interval
G	Gradient
dc	Change in cement content
x_i	Test result NOTE: According to EN 206-1, a test result may be the mean value of two or more specimens taken from one sample and tested at one age.
$\bar{x}$	Mean value of 'n' test results

1 Introduction

It is safe to assume that ever since manufacturing commenced, attempts have been made to control the process in order to improve quality and drive down costs. The application of statistical techniques to manufacturing was first developed by physicist Walter A. Shewhart of the Bell Telephone Laboratories in 1924. Shewhart continue to develop the idea and in 1931 he published a book on statistical quality control [1].

Shewhart recognised that within a manufacturing process there were not only natural variations inherent in the process, which affected quality but there were also variations that could not be explained. Shewhart recognised that it is possible to set limits on the natural variation of any process so that fluctuations within these limits could be explained by chance causes, but any variation outside of these limits, special variations, would represent a change in the underlying process.

Shewhart's concept of natural and special variations is clearly relevant to the production of concrete at a ready-mixed plant or precast factory and the requirement to achieve a specified compressive strength. Natural variations exist in the process due to variation in the raw materials (aggregate grading, chemical composition etc), batching accuracy, plant performance, sampling and testing etc. Special causes of variation, outside of the natural variations could be due to changed constituent materials being used, weigh-scales losing accuracy, a new batcher, problems with testing equipment etc.

Control charts have found widespread use in the concrete industry in both ready-mixed concrete and precast concrete sectors as a tool for quality control. Control charts can be applied to monitor a range of product characteristics (e.g. cube/cylinder strength, consistence, w/c ratio), constituent materials (aggregate grading, cement strengths etc.) or production (batching accuracy).

Their most common application of control charts is as a means of continuously assessing compressive strength results in order to:

- check whether target strengths are being achieved;
- measure the variations from target (all products vary);
- identify magnitude of any variation;
- objectively define action required (e.g. change w/c ratio) to get the process back on target;
- identify periods and concretes where the strength was less than specified, so that investigations can be carried out and corrective action taken.

The use of control charts should not be treated in isolation from the rest of production control. For example routine checking and maintenance of weigh equipment will minimise the risk of a weigh-scale failure. Control charts provide information about the process, but the interpretation of the information is not a mechanical process. All the information available to the concrete producer should be used to interpret the information and make informed decisions. Did a change in quality occur when a new batch of constituent was first used? Is all the family showing the same trend? Are other plants using similar materials showing a similar trend? Such information leads to the cause of the change in quality being identified and appropriate action being taken. For example a loss of accuracy in the weigh-scales should lead to repair, maintenance and re-calibration and not a change in mix proportions. Where a change in mix proportions is required, the use of control charts can lead to objectively defined changes in proportions.

Effective control of concrete production is more easily achieved when there are good relationships with the constituent material suppliers, particularly the suppliers of cementitious materials. Early warning of a change in performance from the constituent material supplier should be part of the supply agreement, e.g. that stock clinker is being used during the maintenance period, and on the basis of this warning, the producer will decide the appropriate action.

Some producers use changes in cement chemistry to predict changes in concrete strength. Effective production control is about using all this information to produce concrete conforming to its specification. Effective production control, which includes the use of control charts, significantly reduces the risk of non-conformity benefiting both users and producers of concrete.

There are drawbacks to the existing method of assessment of conformity of mean strength adopted in EN 206-1 including not following the CEN Guidance on the evaluation of conformity [2]. It is believed that control charts (already widely used as a quality assurance tool in factory production control) would provide an alternative and better means of ensuring the characteristic strength is achieved and it is a method that follows the CEN Guidance.

This publication will review various control systems that are currently used in the concrete industry and, by the use of examples, show how the principles are applied to control the production of concrete.

2 Statistics for Concrete

2.1 Normal distribution of strength

Compressive strength test results tend to follow a normal distribution as illustrated in Figure 1. A normal distribution is defined by two parameters, the mean value of the distribution and the standard deviation (σ), which is the measure of the spread of results around the mean value. A low standard deviation means that most strength results will be close to the mean value; a high standard deviation means that the strength of significant proportions of the results will be well below (and above) the mean value. The area under the normal distribution between two values of 'x' represents the probability that a result will fall within this range of values. The term 'tail' is used to mean the area under the normal distribution between a value, e.g. a compressive strength, and where the frequency is effectively zero. For strength it is the lower tail, i.e. low strength results, that is important but for other properties, e.g. consistence, both the lower and upper tails are important.

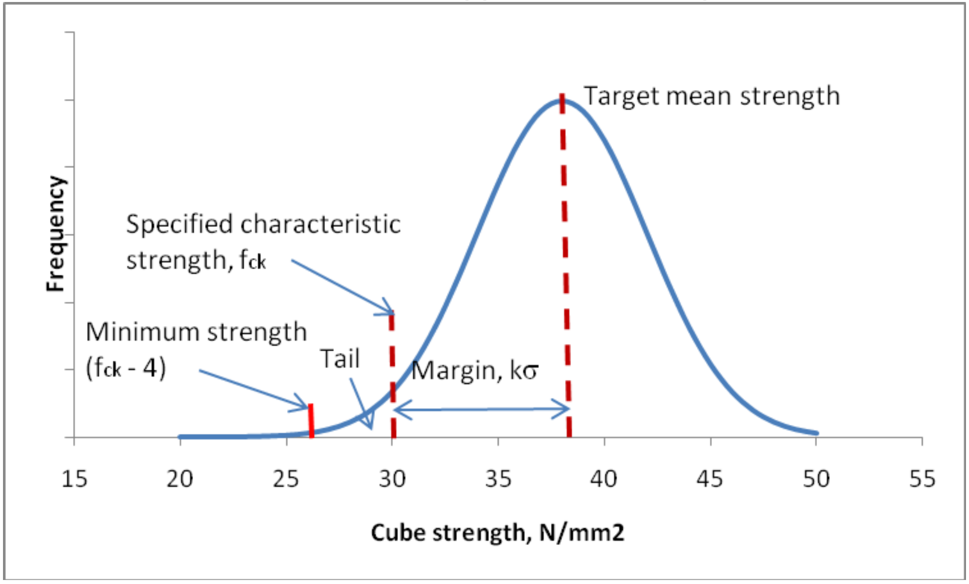

Figure 1: Illustration of concrete strength distribution

At the extremes of the strength range for a given set of constituent materials, the assumption of a normally distributed set of data may not be valid. It is not possible to have strengths less than zero and most concretes have a ceiling strength beyond which they cannot go. In these situations the data set is skewed. However as low strengths are of concern to specifiers, an assumption of normally distributed data does not lead to problems in practice.

2.2 Characteristic strength and target strength

EN 206-1[3] specifies the characteristic compressive strength of concrete in terms of a standard cylinder test or a standard cube test carried out at 28 days. The characteristic strength is defined in EN 206-1 as the "value of strength below which 5% of the population of all possible strength determinations of the volume of concrete under consideration, are expected to fall". Put simply this means that if every single batch was tested, 5% of the results would fall within the lower 'tail' of the normal distribution that starts 1.64σ below the actual mean strength. However the actual mean strength will not be known until the concrete has been produced and tested and therefore the target mean strength (TMS) is usually set at some higher value to ensure the concrete achieves at least the specified characteristic strength.

The target mean strength is given in Equation 1.

$$TMS = f_{ck} + k \times \sigma \qquad \textit{equation 1}$$

Where TMS = target mean strength
f_{ck} = characteristic compressive strength
σ = estimate for standard deviation of population
k = statistical constant

$k \times \sigma$ = the margin

The fixed point in the distribution is the specified characteristic strength and as the margin increases and/or the standard deviation increases, the target mean strength increases, see Example 1.

Example 1
The target mean strength for a specified characteristic strength of C25/30 is given in Table 1. A standard deviation (σ) of 3 N/mm^2 is typical of a concrete with low variability and a value of 6 N/mm^2 represents high variability.

Table 1: Target mean strength for specified characteristic strength of 30N/mm^2 (cube)

Margin	*Area in lower tail (i.e. percentage below characteristic strength)*	*Target mean strength (cube), N/mm^2*	
		σ = 3 N/mm^2	*σ = 6 N/mm^2*
1.64σ	*5%*	*35*	*40*
1.96σ	*2.5%*	*36*	*42*
2.00σ	*2.28%*	*36*	*42*
2.33σ	*1.0%*	*37*	*44*
3.0σ	*0.13%*	*39*	*48*

The numbers in this table have been rounded.

A concrete strength below the characteristic strength is not a failure as statistically 5% of the results are expected and accepted as to fall below this value. However for structural safety reasons, a batch with a concrete strength significantly below the characteristic strength is excluded, even though it forms part of the expected population. Consequently EN 206-1 specifies a minimum strength requirement for individual results (f_{ci}) of (f_{ck} — 4). Any batch below this strength is a non-conforming batch.

The risk of non-conformity decreases as the margin increases. Statistics are used to quantify that risk. For a given margin the probability of a test result falling below the specified characteristic strength or failing the individual strength criterion is given in Table 2. Table 2 shows that the probability of having a result below the specified characteristic strength is independent of the standard deviation (as the margin is based on the standard deviation) but the risk of failing the criterion for individual batches increases as the standard deviation increases.

Table 2: Effect of margin on proportion of concrete below characteristic strength; and risk of failing the strength criterion for individual batches

Margin	Probability of a test result being below the characteristic strength	Risk of failing the strength criterion for individual batches	
		σ = 3 N/mm^2	σ = 6 N/mm^2
1.64σ	1 in 20 (5%)	0.1%	1%
1.96σ	1 in 40 (2,5%)	0.05%	0.4%
2.33σ	1 in 100 (1%)	0.01%	0.1%
3.08σ	1 in 1000 (0,1%)	0.0005%	0.01%

The definition of 'characteristic strength' in EN 206-1:2000 has its complications. For a structural engineer the phrase 'the volume of concrete under consideration' may be applied to all the concrete in their structure and to the concrete in a single element of that structure even if this comprises a single batch. For conformity to EN 206-1, the 'volume under consideration' is all the concrete in an assessment period. Neither of these interpretations of this phrase is suitable for use in control systems as the process is continual. Caspeele and Taerwe [5] have proposed that if the production achieves an average outgoing quality limit[a] (AOQL) of 5%, the production can be accepted as having achieved the characteristic strength.

2.3 Standard deviation

The standard deviation of a population will only be truly known if every batch of concrete is tested. However if 35 or more results are available, the estimated standard deviation is likely to be very close to the true standard deviation. This is the reason why EN 206-1 requires 35 results to calculate the initial standard deviation

When $n \geq 35$, the standard deviation may be estimated using the equation:

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{(n-1)}}$$

Alternatively it can be determined through a range of pairs approach where

Mean range of successive pairs = 1.128 x standard deviation *equation 2*

or,

Standard deviation = 0.886 x mean range of successive pairs of results

The range is the numerical difference between successive results and the difference is always taken as a positive number, e.g. |2-3|=1. The range of pairs method of calculating the standard deviation is particularly suited for populations where there are step changes in mean strength in the data set, e.g. concrete, as the effect of the step change will be limited to a single pair of results. With concrete production, step changes in mean strength (usually

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[a] From the operating-characteristic curve for the selected sampling plan, the average outgoing quality (AOQ) curve is determined by multiplying each percentage of all possible results below the required characteristic strength in the production by the corresponding acceptance probability.

due to a change in a constituent) are more common than drifts in mean strength.

<u>*Example 2*</u>

Table 3: Calculation of the standard deviation using mean range

Result	*Transposed cube strength, N/mm^2*	*Range, N/mm^2*	*Calculation of standard deviation*
1	*54.5*		*Estimation of the standard deviation* *= 0.886 x 51/14* *= 0.886 x 3.64 = 3.0 N/mm^2 (rounded to the nearest 0.5 N/mm^2)*
2	*52.5*	*2.0*	
3	*49.5*	*3.0*	
4	*47.5*	*2.0*	
5	*49.0*	*1.5*	
6	*43.5*	*5.5*	
7	*54.5*	*11.0*	
8	*46.5*	*8.0*	
9	*50.0*	*3.5*	
10	*50.5*	*0.5*	
11	*47.0*	*3.5*	
12	*48.5*	*1.5*	
13	*53.0*	*4.5*	
14	*51.5*	*1.5*	
15	*48.5*	*3.0*	
Sum of ranges		*51.0*	
Mean of ranges		*3.64*	

<u>*Example 3*</u> *(copied from reference [4])*
15 random data have been generated assuming a mean strength of 37.0 N/mm^2 and a standard deviation of 3.5 N/mm^2. These have been repeated to give a total of 30 data, see Figure 2a. The standard deviation of the 30 data given in Figure 2a is:

3.6 N/mm^2 when determined by the standard method;
3.7 N/mm^2 when determined from 0.886 x mean range.

To illustrate the effect of a change in mean strength on the standard deviation, an extreme reduction in mean strength of 5.0 N/mm^2 is introduced at result 16 i.e. data 16 to 30 are all 5.0 N/mm^2 less than in Figure 2a. The dispersion of the data around these mean strengths is unchanged. The standard deviation of the 30 data given in Figure 2b is:

4.4 N/mm^2 when determined by the standard method;
3.8 N/mm^2 when determined from 0.886 x mean range.

This shows that the standard deviation calculated from the mean range has been less affected by the change in mean strength.

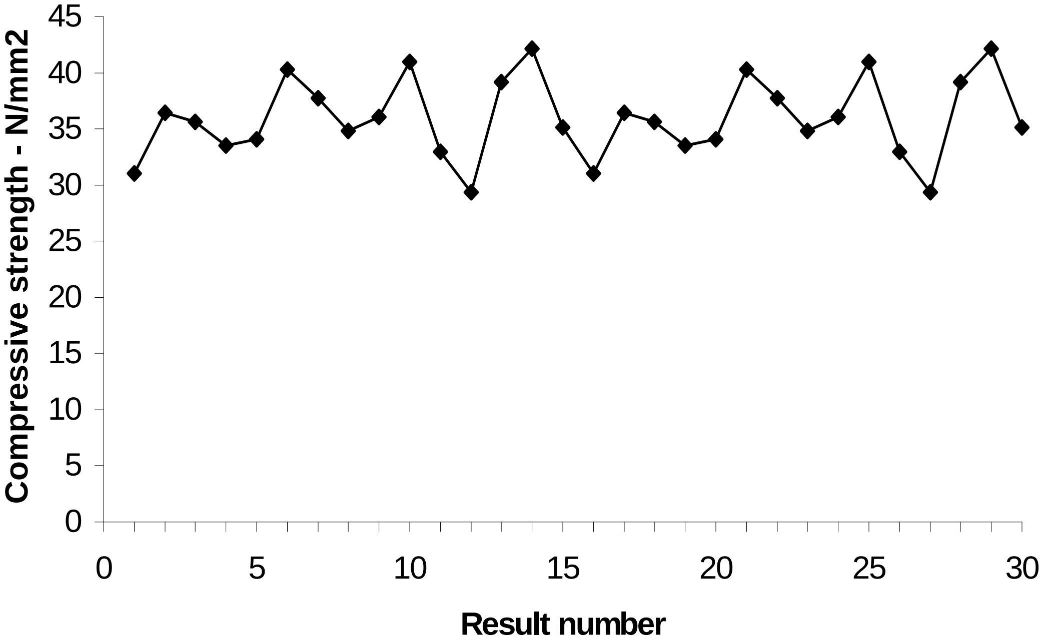

Figure 2a. Fifteen random data generated assuming a mean strength of 37.0 N/mm^2 and a standard deviation of 3.5 N/mm^2 (the first group of 15 results are the same as the second group of 15 results).

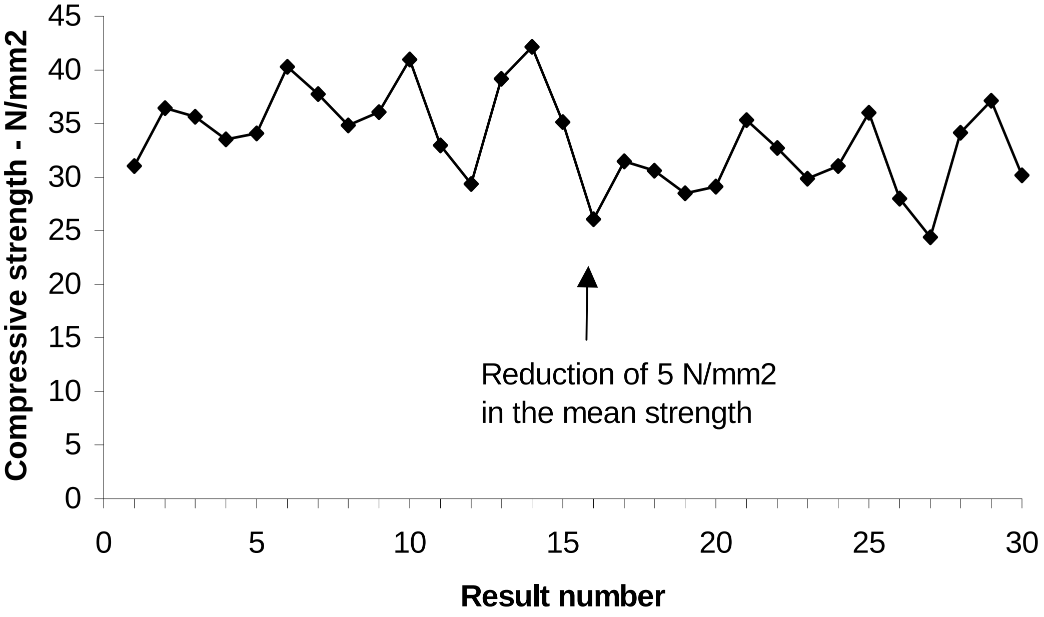

Figure 2b. The same data as in Figure 6a, but with a reduction in mean strength of 5.0 N/mm^2 introduced at result 16.

The true standard deviation of a population, σ, can only be determined if all the population were to be tested, which is impractical. In practice the population standard deviation is estimated by testing samples. The more samples that are tested, the more reliable will be the estimated population standard deviation. EN 206-1 requires at least 35 results to initially estimate the population standard deviation. Prior to obtaining the estimated population standard deviation, concrete is controlled by more conservative initial testing rules. Without an estimated population standard deviation, it is not possible to use control charts to control the concrete production.

Once the initial population standard deviation has been estimated, EN 206-1 permits two methods for verifying the initial estimate. The first method involves checking that the standard deviation of the most recent 15 results does not deviate significantly from the adopted value. The second method involves the use of continuous control systems.

The standard deviation for strength tends to be constant for medium and high strength mixes but for lower strengths it tends to increase proportionally with mean strength [6] and the relationship illustrated in Figure 3 may be assumed. In practice this means that the standard deviation for concretes that have a characteristic strength of 20 N/mm^2 or more is determined by testing and calculation, while the standard deviation for concrete with a lower strength is interpolated.

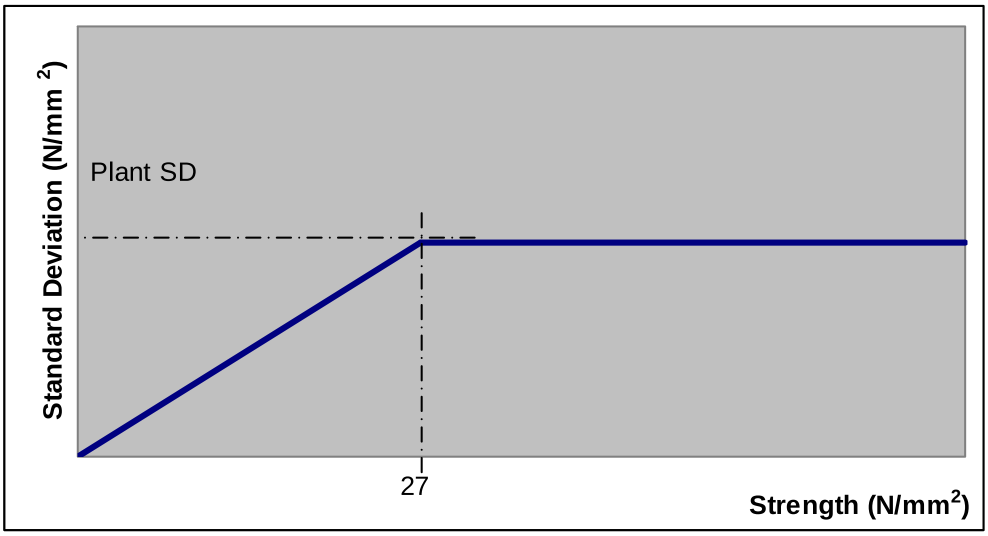

Figure 3: Simplified standard deviation to mean strength relationship

2.4 Setting the target strength

The target strength is set to achieve a balance between the following requirements:

- high probability of achieving a population with at least the specified characteristic strength;
- low risk of failing the minimum strength criterion;
- low consumers risk;
- low producers risk;
- competitive and economic.

The target strength is selected by the producer, but the producer may have to comply with certain minimum values. The target strength should never be lower than (f_{ck} +1.64σ), but it is normally higher than this value. National requirements, the requirements of a certification body or other requirements (see **9.4**) may impose minimum target strengths.

UK experience is that a minimum target strength of (f_{ck} +1.96σ) at a test rate of at least 16 results per month is a good balance between these conflicting demands. With a concrete family this gives about a 3σ margin, i.e. a 1:1000 risk of failing the minimum strength requirement (f_{ck} – 4). Data collected by a UK certification body on individual batch non-conformities shows that the actual rate of non-conformity is an order of magnitude lower and this is due to the active control of the production.

3 Simple Data Charts

Simple data control charts are used to routinely monitor quality. There are two basic types of control charts.

Univariate – a control chart of one quality characteristic (e.g. mean strength)

Multivariate - control chart of a statistic that summarises or represents more than one quality characteristic (e.g. coefficient of variation)

If a single quality characteristic has been measured or computed from a sample, the control chart shows the value of the quality characteristic versus the sample number or versus time.

Simple data charts are useful in providing a visual image of production and unusual results. Simple charts may also give an indication of trends but the general scatter of the data may also mask trends that can be identified only by more in-depth analysis of the data.

Consider the data in Table 4 and illustrated in Figure 4. A review of the data shows that all the results are within +/- 8 N/mm^2 of the target. The results are fairly evenly distributed around the target (2 on target, 9 results above and 7 below) so it is not immediately obvious what conclusions can be drawn from the data.

Table 4: Example data for mean strength with a target strength of 40N/mm^2

Result	28 day strength, N/mm^2	Result	28 day strength, N/mm^2
1	37	10	40
2	42	11	34
3	36	12	44
4	35	13	46.5
5	42	14	42
6	38	15	44.5
7	39.5	16	45
8	40	17	44
9	35	18	48

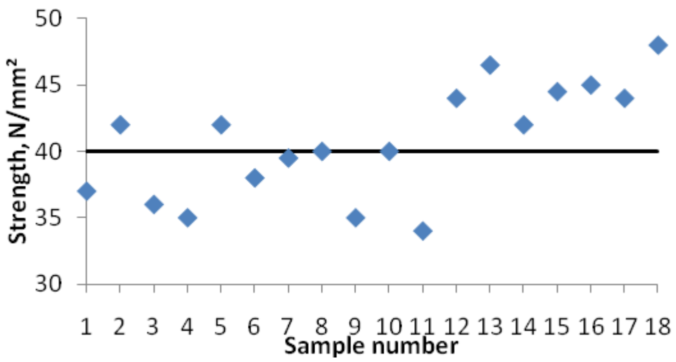

Figure 4: Simple univariate control chart for strength

4 Shewhart Charts

4.1 Introduction

While graphical plots can give useful information about the pattern of a production process, the control chart becomes a much more powerful tool if statistical rules are also applied to the data. Shewhart control systems measure variables in the production processes (e.g. target mean strength).They make use of calculated control limits and apply warning limits based on the measured variation in the production process.

ISO 8258 [7] gives general information on Shewhart control charts and ISO 7966 [8] gives general information on Shewhart control charts for acceptance control.

The Shewhart chart will have a horizontal central line which represents the expected mean value of the test results on the samples taken from production; in the case of concrete, the Target Mean Strength for a chart controlling compressive strength. Lines representing the upper control limit (UCL) lower control limit (LCL), upper warning limit (UWL) and lower warning limit (LWL) may also be added. Generally action is required if a result is beyond either of the control limits.

The UWL and LWL are set at a level so that most of the results will fall between the lines when a system is running in control. These are not specification limits but 'warning' limits based on the variability of the production process. Given that concrete strengths follow a normal distribution (Figure 1), it follows that there is a 50% chance that a result will be above the TMS and a 50% chance that it is below the TMS. In chapter 2 it was shown that a margin of 1.96 x σ will lead to 2.5% of results being below the characteristic strength. Some variables, e.g. consistence, have both upper and lower limits and in these cases it is essential to have both an UWL and a LWL. While for conformity to a specified characteristic strength a high value is not significant, from the viewpoint of economic production it does matter. Therefore in practice, both upper and lower warning limits are used even for a variable that has a single limit value, e.g. concrete strength. Setting upper and lower warning limits at 1.96σ leads to the expectation that 95% of the results will fall within these limits and 2.5% in each of the 'tails' of the normal distribution. If a margin of 3.0 x σ is adopted, there is very little chance of a result falling outside this limit due to natural variation (0.3% for two-tailed test). A Shewhart control chart can be constructed with

UCL = TMS + 3 x σ
LCL = TMS – 3 x σ
UWL = TMS + 2 x σ
LWL = TMS – 2 x σ

The probability of a single result falling outside of either the UWL or LWL is 4.56%, i.e. 2.28% above the UWL and 2.28% below the LWL (see Table 1*Table 2*).

The probability of two consecutive results falling outside the limits purely by chance is

= 0.0456 x 0.0456 = 0.002079 or 0.21%

The probability that the two results are either both above or below the line (i.e. in the same direction) is only 0.05%. Such an outcome is very strong evidence that the expected outcome is not being achieved.

4.2 Shewhart action criteria

4.2.1 Points beyond UCL or LCL

The presence of one or more points lying outside of the UCL or LCL is primary evidence that the system is out of control at that point. Since there is only a 0.3% chance that this result is due to natural variation, it is probable that special variation will account for the extreme value and an immediate investigation into the cause should be undertaken.

4.2.2 Points beyond UWL or LWL

The presence of two consecutive, or more than 1 in 40, points beyond either warning line is evidence that the process is out of control and an investigation of the data should be undertaken.

4.2.3 Patterns within control limits

It is also possible to analyse data that doesn't breach either the control or warning limits to evaluate whether any trends are significant. Runs analysis can give the first warning of a system going out of control before points are seen beyond the warning limits.

The following simple rules of thumb have been proposed for sequences of results that remain within the warning limits [9]:

1. Seven or more consecutive results on the same side of the target mean strength
2. At least 10 out of 11 results on the same side of the target mean strength
3. At least 12 out of 14 results on the same side of the target mean strength
4. At least 14 out of 17 results on the same side of the target mean strength

4.3 Control of standard deviation

The control and warning limits are determined by the standard deviation of the process; it is therefore important to monitor the standard deviation. As the calculation to determine standard deviation is relatively complex, the alternative calculation in *equation 2* is used linking standard deviation to the range of pairs of results. Plot the running mean range of the last n successive results where $n \geq 15$ against test result number. Select the change in standard deviation that will prompt action ($\Delta\sigma$) and set action lines at:

$$1.128\text{*current standard deviation} \pm 1.128\text{*}\ \Delta\sigma$$

4.4 Example Shewhart chart

Consider again the strength data in Table 4 and subject it to a Shewhart analysis using the rules stated in **4.2**. Figure 5 shows the data with a UCL, LCL, UWL and LWL applied. Immediately it is apparent that point 18 has exceeded the UWL This does not breach the rule defined in **4.2** (requiring 2 consecutive points above UWL) but also at this point there is a sequence of 7 points on the same side of the target mean strength (see **4.2.3**). The Shewhart chart is showing that the process is out of control, i.e. the actual mean strength is higher than the mean strength required.

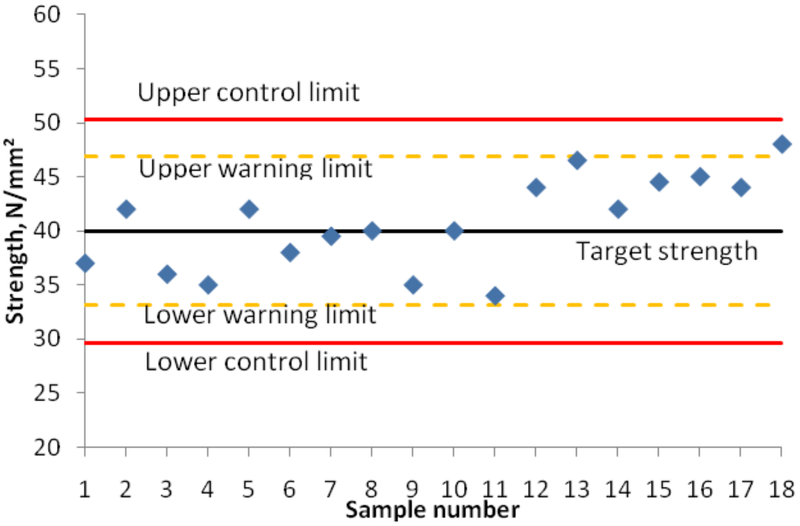

Figure 5: Control levels applied to data

4.5 Modified application of Shewhart control chart

If the aim is to assess whether the level of production is higher than a specified characteristic value, a modified application of Shewhart control charts can be used, with the use of specific modified variables. This application comprises checking that the average of *n* measured strength results is greater than a lower line L_l situated at a given distance from f_{ck} with the following variables:

$L_l \geq f_{ck} + (q_n \times s)$ where

- q_n depends on *n* and on the AOQL chosen,
- s is an updated evaluation of the standard deviation of the relevant production.

In the case where $n \geq 15$ and $q_n \geq 1.48$, the Shewhart charts will satisfy the requirement for an AOQL of 5%. This criterion also satisfies the conformity criteria for mean strength in EN 206-1. A warning line at some higher value may also be appropriate.

Example 4

A precast concrete factory intends using a Shewhart chart system to show conformity to the mean strength criterion in EN 206-1. Due to process requirements the strengths tend to exceed the characteristic strength within a few days and therefore they opt to test at a real age of 7 days to verify that the specified 28 day strength is already being achieved at 7 days. As the compressive strengths are expected to be well above the specified strength, they opt not to have a warning line.

To do this task the running average strength of the last 'n' consecutive results, where 'n' is a predetermined number that is at least 15 are plotted on a Shewhart chart with one limit line with a value of (f_{ck} + 1.48s). If the running mean strength below this line this indicates that an AOQL of 5% is not being achieved. A warning line at some value higher than (f_{ck} + 1.48s) may be added.

The specified compressive strength class is C25/30 and they use cubes for assessing the production and conformity. The current standard deviation is 2.5 N/mm^2. The limit value LI is:

*30 +1.48*2.5 = 33.7 N/mm^2*

For control purposes, rather than using non-overlapping groups of results, they opt to use the running mean of the last 15 results. This is shown in Figure 6a), which shows that the mean strength is consistently above the limit value.

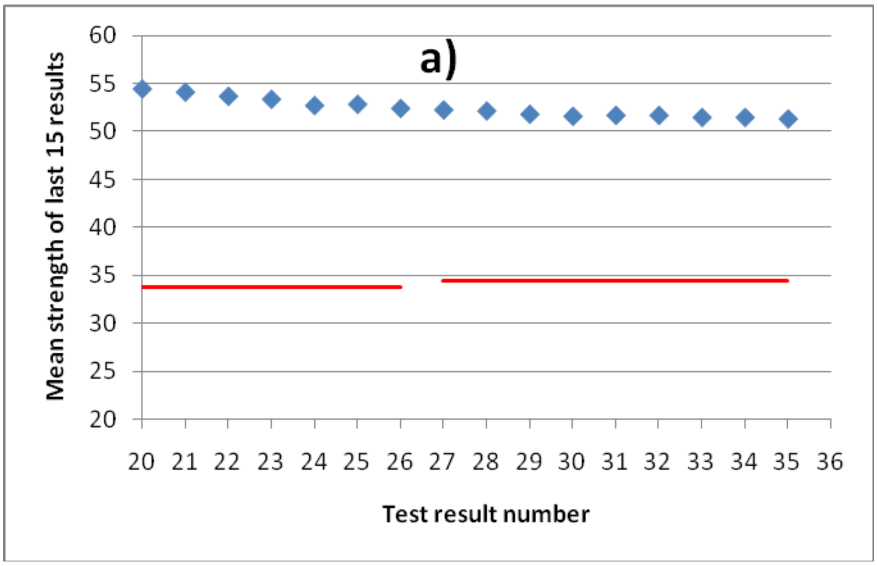

Figure 6a): Control of conformity of mean strength using a running mean of 15 results

It is also necessary to check that the sample standard deviation has not changed significantly. Though clause 8.2.1.3 of EN 206-1:2000 states that the present value is still applicable if the calculated value based on the last 15 results is within about ± current value/3, this is not a very sensitive indicator of change and most producers would regard a 0.5 N/mm^2 change in standard deviation as significant and the precast company uses this value of 0.5 N/mm^2, which is *controlled with another modified Shewhart chart. On this chart a horizontal line is drawn at the current mean range value (1.128σ) and action lines ± 1.128 x 0.5 from this value.*

Again a running mean range of the last 15 consecutive and overlapping pairs of results is used. When the running mean value crosses one of these action lines, this indicates that the standard deviation has changed by 0.5 N/mm^2 and a new value should be applied.

In this example the current standard deviation is 2.5 N/mm^2 and this equates to a mean range of 1.128 x 2.5 = 2.82 N/mm^2 and upper and lower action lines at 3.38 N/mm^2 and 2.26 N/mm^2 (2.82 $\pm$ 1.128 x 0.5). These are shown in Figure 6b). At test result 26, the mean range crosses the upper action line indicating *that the standard deviation has increased by 0.5 N/mm^2. The limit value is increased in Figure 6a) to 34.4 N/mm^2 and in Figure 6b) a new mean range is set at 3.38 N/mm^2 with upper and lower action lines set at 3.94 N/mm^2 and 2.82 N/mm^2 respectively. As the running mean strength is still well above the limit line, the mix proportions are not changed, i.e. the appropriate action is to take no action other than change the values on the control charts.*

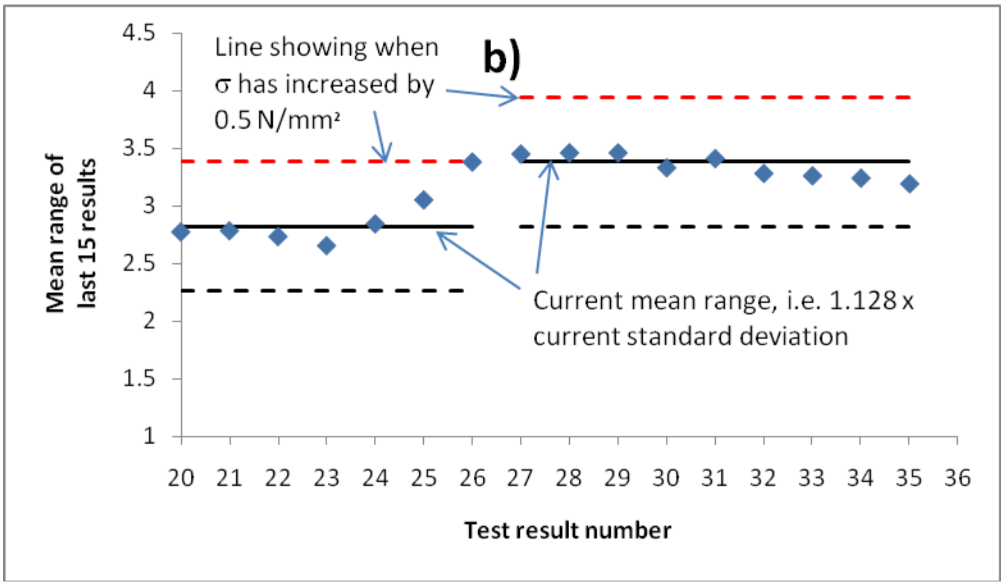

Figure 6b): Control of standard deviation using the running mean range of the last 15 ranges

5 CUSUM

5.1 Introduction

CUSUM control systems (short for cumulative sum) were developed in the 1950s, initially for quality control of continuous manufacturing processes. They have found widespread use in the concrete industry. In CUSUM charts, the central line does not represent a constant mean value but is a zero line for the assessment of the trend in the results. In concrete production three CUSUMs are used:

- CUSUM M, for the control of mean strength;
- CUSUM R (range), for the control of standard deviation;
- CUSUM C, for the control of correlation.

The CUSUM method, described in more detail in BS 5703[9] and Concrete Society Digest No. 6[10] and ISO/TR 7871[11], involves subtracting the test result from a target value then producing an ongoing running sum (the CUSUM) of the differences. If the process is in control, the points on the CUSUM plot are distributed randomly (positive and negative differences cancelling each other out), to give an accumulative sum that is close to zero, but if the process slips out of control, this will be quickly illustrated by the CUSUM plot moving towards the UCL or LCL.

BS 5700[12] describes the following advantages of the CUSUM system:

a) for same sample size it gives a more vivid illustration of any changes;
b) uses data more effectively therefore produces cost savings;
c) gives clear indication of location and magnitude of change.

CUSUM charts have been found to be more sensitive at detecting small shifts in the mean of a process than Shewhart, whereas Shewhart charts are superior at detecting large shifts [13]. When the CUSUM reaches the UCL or LCL, it is possible to use the plot to determine at what point the process went out of control and what scale of corrective action is required.

Historically, CUSUM control charts were plotted manually and to determine whether a trend in the plot was significant or not, a transparent mask in the shape of a truncated (cut-off) V is placed on its side over the plot, with the lead point placed on the most recent result, not on the central line. The transparent V-mask overlay in Figure 7 is shown in red. Each limb is marked with its standard deviation and in this example the values run from 3.0 N/mm^2 in 0.5 N/mm^2 steps to 5.0 N/mm^2. The limbs have no limitation on their length. The ‘arms’ of the V-mask represent the upper and lower control limits. If the plot crossed either the upper or lower arm of the V, a significant change is deemed to have occurred, see Figure 7. Computerized systems have taken over the analysis of CUSUMs but the concept is easier to understand using the more visual V-mask method.

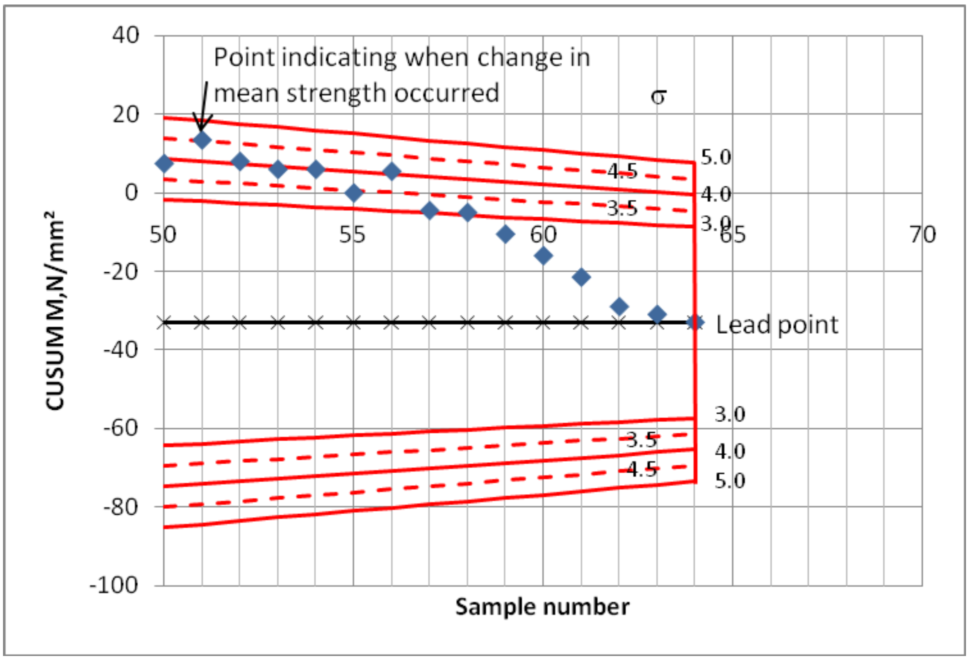

Figure 7: Illustration of V-mask placed over lead point to detect change (the current standard deviation is 4.5 N/mm^2)

Consider the data for compressive strength in Table 4 for a plant operating on a standard deviation of 3.5 N/mm^2 and with a target mean strength of 40 N/mm^2. The CUSUM can be calculated and is tabulated in Table 5. A plot of the CUSUM with UCL and LCL is shown in Figure 8. When the V-mask is placed on the lead point (point 18), the CUSUM crosses the LCL at point 11 indicating that a change in the process occurred at this point. While the Shewhart analysis of the data (see **4.4**) also indicated a change at point 18, the plot of the CUSUM gives a clear visual picture of the trend and shows that it has been present since point 11.

Table 5: CUSUM data

Result	28 day strength (N/mm^2)	Difference from 40N/mm^2 target (N/mm^2)	CUSUM (N/mm^2)
1	37	-3	-3
2	42	2	-1
3	36	-4	-5
4	35	-5	-10
5	42	2	-8
6	38	-2	-10
7	39.5	-0.5	-10.5
8	40	0	-10.5
9	35	-5	-15.5
10	40	0	-15.5
11	34	-6	-21.5
12	44	4	-17.5
13	46.5	6.5	-11
14	42	2	-9
15	44.5	4.5	-4.5
16	45	5	0.5
17	44	4	4.5
18	48	8	12.5

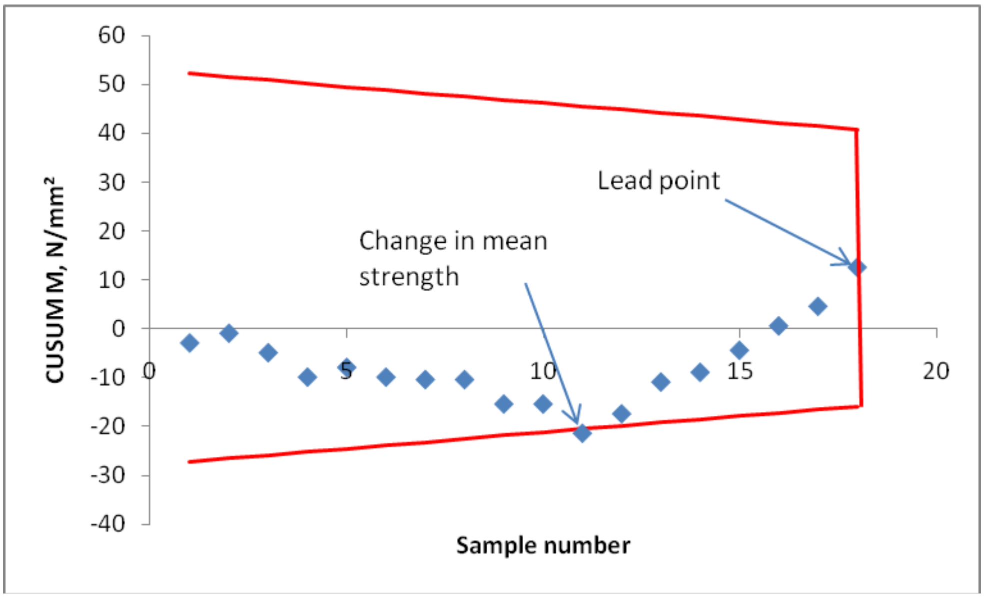

Figure 8: Example CUSUM plot for data in Table 5

5.2 Controlling mean strength

Compressive strength of concrete is the simplest parameter to monitor. The test result is compared with a target mean strength based on the specified characteristic strength plus the margin (see **equation 1**).

CUSUMs controlling compressive strength are generally based on a concrete family and all test results are converted to the equivalent value of a selected reference concrete, e.g. C32/40. The analysis is usually based on early age test results (see **7.1**), e.g. on 7 day strength data, as the risk of waiting until 28 days to identify a loss of control is unacceptable. The predicted 28 day strength is calculated from the early age test result (see **7.1**) and this is used in the CUSUM until the actual 28 day strength is available. Some systems then substitute the measured 28-day strength and re-calculate the CUSUM while other systems continue to use the predicted strength. The current trend is to replace the predicted strength with the measured 28 day strength as since 2000 the data have also been used for conformity and conformity is based on the measured 28 day strength.

A worked example of a CUSUM controlling target mean strength (CUSUM M) using the concrete family approach is detailed in chapter **11**.

5.3 Controlling standard deviation

CUSUM may be used to monitor standard deviation by using the relationship between the range of successive pairs of results given in *equation 2*.

The expected range (current standard deviation x 1.128) of a pair of results is monitored against the actual range and a CUSUM of the differences calculated. The results are plotted and monitored using a V-mask or computer model.

It should be noted that range is not a normally distributed variable, therefore if a symmetrical V-mask is adopted; longer run lengths will be required to detect a decrease in the standard deviation. However this conservative approach – the use of a symmetrical mask- is adopted for simplicity in many systems.

When a significant change is detected, the standard deviation is adjusted and results corrected to the new target mean strength – higher if the standard deviation has increased, lower if the standard deviation has decreased. The result immediately prior to the change of target strength should also be adjusted to the new target mean strength to calculate the range of pairs, otherwise an additional element of variation would be built into the data.

An example CUSUM on standard deviation (CUSUM R) is illustrated in chapter **11**.

5.4 Controlling correlation

Control systems for concrete are generally based on early age strength as the consequences of waiting for the 28 day compressive strength results lead to an unacceptable level of risk. Seven day compressive strength results are normally used in the control system and the 28 day strength is predicted using an estimated correlation (the relationship between 7 and 28 day strength). In order to confirm that the correlation factor is correct, a CUSUM C may be run on the differences between actual and predicted 28 day strengths. If the

CUSUM C is positive then the system is underestimating the 28 day strength and if negative it is overestimating the 28 day strength.

When a significant trend is detected, a new correlation relationship is determined. The CUSUM M for mean strength using predicted results will need to be recalculated as the system has effectively been under or over estimating for a period of time and it may be significantly adrift. The plot of range need not be re-determined because the correlation change will affect all results similarly (except for the range straddling the point of correlation change).

The relationship between 7 and 28 day strength is affected by the cement or cement/addition types and sources, e.g. the strength gain between 7 and 28 days of a concrete made with a CEMI cement will be less than for an equivalent concrete[b] made with a CEMIII/B cement. Concretes with different cement/addition types should therefore be either controlled by separate control systems or the difference in correlation between different cement/addition types considered in the corrections that are applied within the concrete family [14]. Different sources of the same type and class of cement or addition may have different 7:28 day strength ratios and if a source of cement or addition is changed, the validity of the current 7:28 day strength ratio should be reviewed. Retarding admixtures might also affect the 7:28 day strength ratio.

It is important to appreciate that calculating the correlation as a simple straight-line relationship or as a simple percentage addition will not be accurate at the limits of the mix design. This is because concrete mixes have ceiling strengths that may be due to a failure of aggregate bond, failure of the aggregate particles themselves or, as the cement content increases, the voidage is increasing in proportion, i.e. the (w + air)/c ratio remaining constant. Therefore at the ceiling strength, increasing the cement content of the mix or using plasticisers to reduce w/c ratio will have little effect on compressive strength. As the 28 day strength approaches the ceiling strength, the ratio between 7:28 day strength will change. Clearly as the 28 day strength approaches the ceiling strength, there will be a reduced strength gain from 7 to 28 days. Similarly, at lower strengths, the results will tend to converge towards zero.

The initial correlation may be established in different ways including:

- Initial trial mixes at a range of cement contents;
- Historical data.

Once the initial correlation is established, it needs to be checked routinely to check that it is still valid. This is the purpose of CUSUM C.
An example CUSUM for correlation (CUSUM C) is shown in chapter **11**. There are fewer data in the correlation CUSUM compared to the CUSUMs for mean and standard deviation because of the 21 day gap between 7 and 28 day results becoming available.

In the example, the actual 28 day results have been used in place of the predicted 28 day data when they become available. Some systems will only use

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[b] A concrete designed to produce the same 28 day strength

predicted data. However, all CUSUMs will recalculate the predicted data when the CUSUM shows a change in the correlation.

5.5 Design of V-mask

The design of the mask (i.e. the appropriate gradients and decision intervals) is based on statistical probabilities and they are linked to the standard deviation of the plant as illustrated in Figure 9 for a typical CUSUM system.

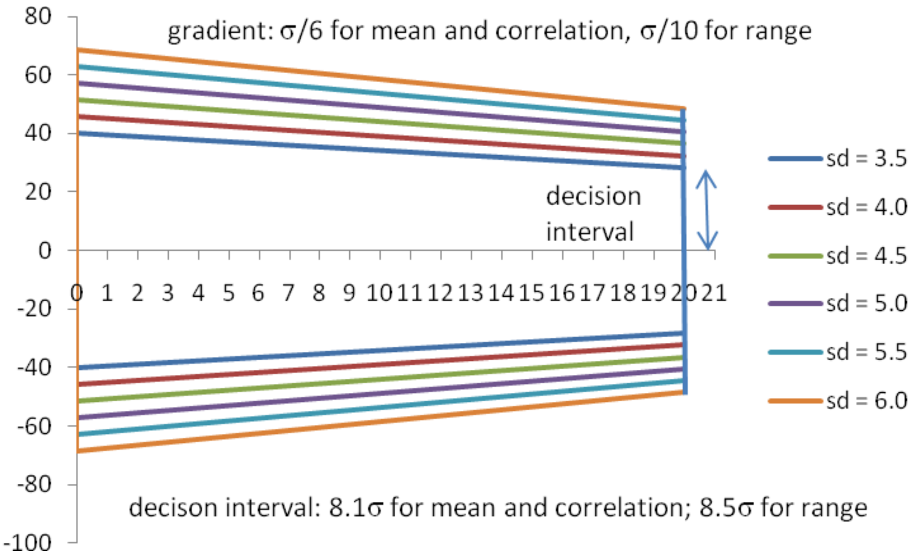

Figure 9: Design of V-mask

A number of concrete producers have opted with their CUSUM systems to follow the 'double-decision approach' used in Shewhart charts, namely a warning line and a control limit, and take action when the warning line is crossed. In this case the warning lines are parallel to the X-axis and offset by the same decision interval as used in the traditional V-mask. Such an approach results in action being taken sooner than with the traditional V-mask. As action is being taken when the warning line is crossed, a control limit serves no practical purpose and they are usually omitted.

5.6 Action following change

When the CUSUM indicates that the target mean strength or assumed standard deviation is not being achieved, adjustments will be made to the concrete production. Often, but not always, this requires a change in cement content or w/c ratio.

The magnitude of the change required when the CUSUM M indicates need for action is effectively determined by the number of results over which the process went out of control (a few points would give a steep gradient and represent a large change, while conversely many points would indicate a gradual change and a shallow gradient).

The change in cement content can be calculated using the parabolic equation:

$$dc = 0.75 \times C_{mra} \times [(DI/n) + G]$$

equation 3

Where:

- dc = change in cement content in kg/m^3
- 0.75 is an anti-hunting factor to prevent oscillatory and complementary changes occurring, i.e. it minimises the risk of over-correcting and having to apply a second adjustment in the opposite direction
- C_{mra} is the relationship between strength and cement content from the main relationship (usually in the range 5 to 6 kg/m^3 to give a strength increase of 1N/mm^2)
- DI is the decision interval of V-mask (see Figure 9)
- G is the gradient of V-mask (see Figure 9).

Both DI and G are a function of standard deviation and have units of N/mm^2.

This relationship is shown graphically in Figure 10 for a change in the CUSUM M: the smaller the number of results the greater is the change required in cement content.

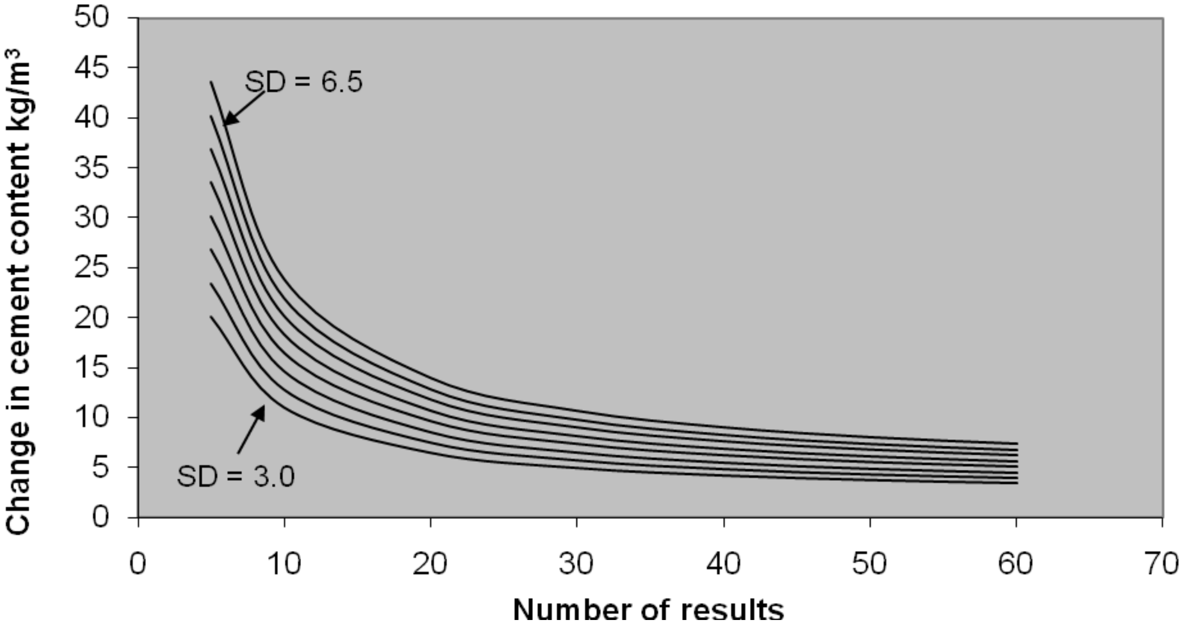

Figure10: Action following change in mean for different SD and run length

A change in the standard deviation would result in a new target mean strength being adopted and therefore a corresponding increase or decrease in the cement content of the control mix and all other mixes in the concrete family.

The information from a CUSUM that a change has occurred is only part of the information available to the concrete producer. The aim of the producer is to identify the cause of the change and take appropriate action at all relevant plants. For example if the change is identified as being due to a change in constituent A, the CUSUMs of all plants using constituent A should be checked to show if they are showing the same trend. If so it would be prudent to take action even if the CUSUMs at one or more of these plants have not yet indicated a change sufficient to cross the control or warning lines.

A change in cement content or w/c ratio is not always required. For example if the cause of the change is known and it has been corrected, adjustment of the mix proportions may not be needed. A typical example of this is where the cement strength is known to have gone down, but subsequently recovered.

6 Multivariable and Multigrade Analysis

Multigrade, multivariable CUSUM was developed by Ken Day [15] as a development of the univariable CUSUM system discussed in chapter 5.

6.1 Multivariable

Multivariable CUSUM systems not only monitor a single property of concrete, e.g. compressive strength, but instead simultaneously monitor a number of properties, for example the Day's CoNAD system plots;

- Compressive strength;
- Density;
- Temperature;
- Slump.

Early action can be taken before the plot reaches the UCL or LCL because the downward trend in compressive strength may be accompanied by a downturn in density and either (or both) an upturn in slump or temperature. If this pattern occurred over two or three results it indicates a genuine change and a probable cause (water addition). The temperature, slump and density information is available immediately and therefore trends can be detected prior to the 7 day strengths being available. An illustration (without control limits) of a plot of multivariables is shown in Figure 11.

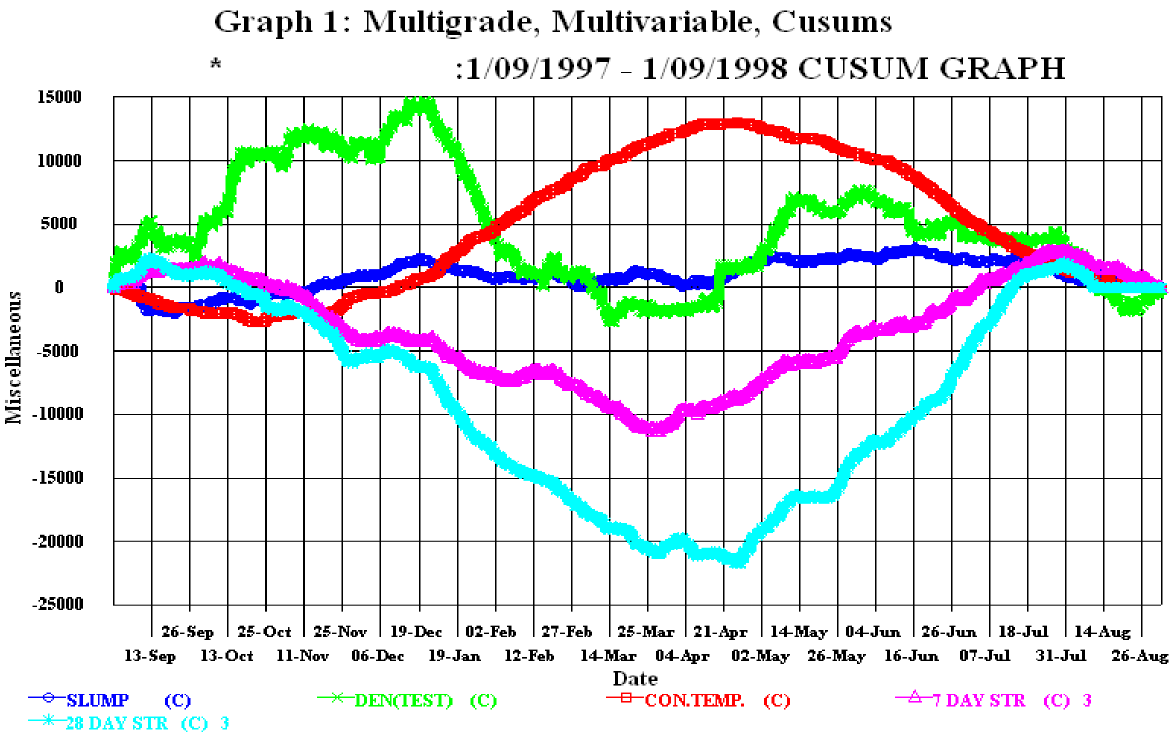

Figure 11: Plot from Multivariable system

6.2 Multigrade

Multigrade CUSUMs are designed to incorporate concretes from different compressive strength classes in the same CUSUM system. One approach is to adopt the family of mixes concept discussed in **7.2**. However Day argues that it is possible to sum differences from the current average value of any variable in each individual strength class of concrete, as though they were all from the same average value. In this way results from high or low strength mixes and even lightweight aggregate mixes can be incorporated into the CUSUM [16].

Day argues [17] that the advantages of this technique are:

1. There is essentially no limit to the range of individual mixes that can be treated as a single family.
2. There is no requirement for adjustment formulas.
3. There is no requirement for checking that constituent mixes remain as acceptable family members (except where a change point is detected).
4. As a consequence of the above, change point detection is much more rapid and multi-variable CUSUMs become more effective in cause detection.

An overall CUSUM can be plotted incorporating all the results. Additionally, the data for individual compressive strength classes can be analysed to determine whether effects are across the range of mixes or are concentrated in specific compressive strength classes.

As the trend is to hold all information on the computer system and interrogate it for trends, differences between the operational practice of using CUSUM and multivariable and multigrade analysis are less than they appear. In both systems, the key is the operator understanding the information being presented by the control system, interrogating the data to determine the cause and taking appropriate action.

7 Speeding the Response of the System

7.1 Early age testing

By using early strength data to predict 28 day strength, an adverse trend will be detected more rapidly. If seven day test data are used, an adverse trend will be detected three weeks earlier than waiting for 28 day strength data.

Generally control systems for mean strength are based on 7 day strengths as the risk of waiting for 28 day test results is unacceptable to both the consumer and producer. However with certain precast elements, process requirements lead to strengths that exceed the specified characteristic strength within a few days and in this case testing may be undertaken at a single age less than 28 days and the process controlled using Shewhart charts. In this case there is no need to predict 28 day strengths as they already exceed what is specified and the sophistication of the CUSUM system is an unnecessary complication.

Systems utilising accelerated tests (e.g. 1 or 2 days) have had limited success [6]. These systems generally entail heating the concrete samples but this process introduces a greater variability than is found in the normally cured 7 day results.

Testing standard cured specimens at three days may introduce greater variability than testing at 7 days. This is because at three days the initial temperature of the concrete, which is controlled by the ambient conditions, has a greater impact on three day strength compared to 7 day strength. In climates that are similar to the UK, the 7:28 day strength ratio is likely to be more stable than a 3:28 day strength ratio and it is the better choice for the control system. The complications of modern concretes, e.g. additions, admixtures, are also best handled using a 7:28 day strength relationship.

The ratio between the early strength test data and the 28 day test data for the same mix is also monitored by a CUSUM to detect any significant change in the relationship. This is the CUSUM C described in **5.4**. However, if one of the other control systems is used with early strength data, it is necessary to continually check that the correlation has not changed. This may be done on a continual basis using, say, the last 10 results or periodically, say, every fifth result.

The multivariable CUSUM approach utilising data from temperature, density and slump (discussed in chapter **6**) claims to allow early interpretation of data, i.e. a detection of a change in quality, than is possible with to a single variable CUSUM.

While predicted 28 day strengths are used to determine if action is needed to keep the process under control, any conformity assessment is based on the 28 day strength data and not the predicted data.

7.2 Family of mixes concept

Changes in the performance of concrete are usually the result of a step change in a constituent material, e.g. between deliveries. Such a change will affect all concrete made with that constituent. A change in the functioning of a plant will affect all concrete produced in that plant. To detect an adverse change in quality requires sufficient data, as a real change has to be separated from normal variability. Controlling each concrete separately will delay the time taken to detect an adverse change. Utilising the family of concretes approach enables more test results to be incorporated into the system. This speeds up the time taken to detect a significant change. For example Figure 5, the Shewhart analysis of the data in Table 4, shows 7 results above the target value and this indicates a significant trend. To identify this trend clearly it needs 7 results to be entered into the system. If each compressive strength class of concrete were monitored separately, it is possible that these 7 results could have been at 7 different compressive strength classes and therefore each control system would only have a single result entered, and no trend would have been detected.

Clearly not all the concrete produced at a ready-mixed concrete plant is the same compressive strength class; in fact a huge variety of mixes can be produced. Concretes can be produced with various compressive strength classes, with different consistences; with and without a variety of admixtures on their own or in combination; with different aggregate sizes, types and proportions of fine aggregates; with different minimum cement content and maximum water cement ratio specifications; with or without fibres; with different cements and additions etc. The range of mixes produced at a precast factory will be significantly smaller. Even with only two concrete types being tested at the same rate, combining the data into a family will halve the time taken to detect an adverse change.

Each mix may be controlled individually (particularly at a precast works), or by the specified compressive strength class. Alternatively, EN 206-1 allows similar concretes to be grouped together into a family[c] of mixes and the control system can be applied to the family.

Annex K of EN 206-1:2000 recommends where there is little experience of the use of concrete families the following may be included in a family

- cement of one type, strength class and source;
- demonstrably similar aggregates and type I additions;
- concretes with or without a water reducing/plasticizing admixture;
- full range of consistence classes;
- concretes with a limited range of strength classes.

Valid correction factors are established for these variables and for the various members of the family, to make results comparable, and to enable them all to be used in a single CUSUM control system.

The starting point for any family is the selection of a consistence, maximum aggregate size and a set of constituent materials. For this selection the relationship between strength and cement content (or water/cement ratio) is determined and this is called the main relationship[d]. The main relationship is usually established by a series of trial mixes at varying cement contents,

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[c] Defined in EN 206-1[3] as "group of concrete compositions for which a reliable relationship between relevant properties is established and documented"

[d] Defined in the QSRMC regulations [18] as "The relationship established between cement content and strength. It will normally be the relationship arising from fitting a curve to data arising from a series of trial concrete mixes"

computer modelling plus a limited number of trial mixes or be determined by analysing production data. From this relationship, the mix proportions to achieve any target strength can be interpolated for this set of materials, consistence and maximum aggregate size. Simple numerical adjustments to the cement and fine aggregate contents (called secondary relationships) are made to convert concretes covered by the main relationship to other concretes in the family or vice versa. The reference concrete to which all other tested concretes are converted lies on the main relationship.

The main and secondary family relationships are used in two directions. They are used to determine the mix proportions needed to satisfy a concrete specification, i.e. for concrete mix proportioning, and they are used to convert the strength of a tested concrete to the value it would have been if it had been the reference concrete. These equivalent strength values are used as input data to control charts.

Where reliable relationships are established to the reference concrete, other concretes may also be included in the family. With the power of computing, the trend is to make use of the option of either including or excluding a wide range of mixes from the family and to analysis the data in different ways. The multigrade system and CUSUM system are capable of supporting such sophisticated analyses. For example a number of plants using the same materials could be included within a family, but while the combined data are analysed for trends, each plant is also reviewed separately. Even when data are not combined into families, the data can be examined to confirm trends. For example if Family A is indicating a particular trend and it is believed to be due to a change of constituent X, the data from other families/mixes containing this constituent should also be reviewed for signs of the same trend and, if appropriate, action.

In general air entrained concrete is not included in a family and it is controlled as a separate concrete, because of the additional variability caused by variations in the air content. However if it is being controlled by a CUSUM system, it is also normal to adjust the strength on the basis of the measured air content. This is to avoid making unnecessary changes to the mix proportions when the changes in strength are simply the result of variable air content.

When controlling a family of mixes, the control strength of the reference concrete (to which other compressive strength classes of concrete are corrected) is either a concrete which represents the most common mix produced at a plant or is mid-range in the family. It has to be a concrete on the main relationship. A number of adjustments may need to be applied to a test result to ensure that it can be compared to the reference concrete, e.g. for slump or for the inclusion of a plasticiser. The first steps are to convert the measured strength of the tested concrete into an equivalent strength of a concrete on the main relationship and then this equivalent strength is moved along the main relationship to become the equivalent strength of the reference concrete. See the CUSUM example in chapter **11**.

In practice, the adjustments for consistence and constituents are applied to the cement content of the tested concrete so that it is deemed to be equivalent to a concrete on the main relationship in terms of constituents and consistence. The measured strength of the tested concrete may be above, below or the same as the strength value on the main relationship for the adjusted cement content. A further adjustment needs to be made to the predicted test result to allow for the difference between the strengths at the adjusted cement content and the cement content of the reference concrete. This is in order to determine what strength the tested concrete would have achieved if it had had the same cement content as the reference concrete. It is this twice adjusted result that is compared with the target mean strength of the reference concrete, and the difference between these values is then applied in the CUSUM.

8 Guidance on Control Systems

8.1 Abnormal Results

From time to time, results will occur that are outliers to the bulk of the data. These results could be due to a number of reasons, for example, a testing error or misreporting. If an outlier is included in the main data analysis, it could disproportionately affect the control system and result in actions being taken to adjust the process that are unnecessary (which could increase the risk to the user of the concrete). However, except where it is technically justifiable and documented, each outlier is checked for conformity against the EN 206-1 individual batch criterion.

A general rule adopted is that results more than 3σ (3 standard deviations) away from the mean should be considered as outliers and therefore excluded from the analysis. The 3σ value equates to 3 results in a thousand (see **Table 2**). However if a result that breaches the 3σ limit is followed by one that is greater than 2σ in the same direction, then both results should be included in the control system and an immediate investigation undertaken to identify whether a significant change has occurred.

8.2 Handling mixes outside the concrete family

The control system will provide information on concrete in the family of mixes that are used within the system. The main relationship for a concrete family can also be used to control mixes that are outside the family. This can be done in a number of ways:

- The cement content of non-family concrete can be linked to a conservative value for that of the control mix, and if the cement content for the control mix is increased, a review is undertaken of the mix design of the non-family concrete;
- Establishing a safe cement content differential (may be higher or lower) to be applied to different cement types or cement blend levels. These differentials can be established through trial mixes in the laboratory or from other control systems. This procedure is particularly useful when a critical concrete is produced infrequently. In effect this increases the margin between the specified characteristic strength and the target strength.

8.3 Handling mixes not controlled by compressive strength requirements

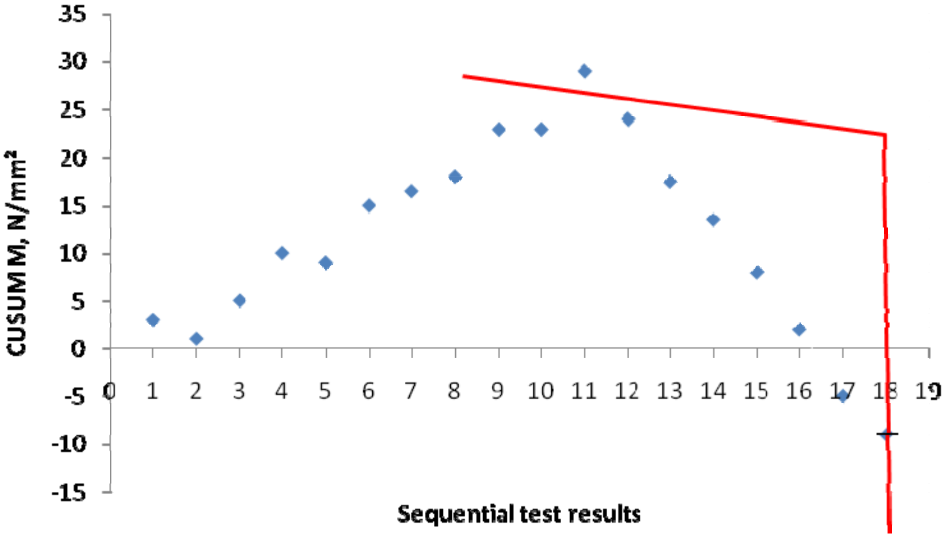
Where mix proportions, particularly cement content per cubic metre, are controlled by the strength criteria, setting the target strength for the mix is relatively straightforward. It is the specified characteristic strength plus the margin. However, strength does not always determine the mix design. For example a foundation to be placed in aggressive ground conditions may require a structural strength of only C25/30, but due to the need to resist the aggressive ground the maximum w/c ratio may be 0.45, which in reality will produce a strength well above that needed for achieving the specified characteristic strength. The target strength should be determined from the higher of the (possibly) different cement contents required to conform to the specified strength, maximum water/cement ratio, and minimum cement content.

If this procedure is not followed, i.e. if the target strength is determined from the specified strength alone, the standard deviation will increase significantly and the result may appear as an outlier (on the high side). However the most serious consequence is that an adverse trend may take much longer to cross the V-mask due to the step change in trend caused by the result.

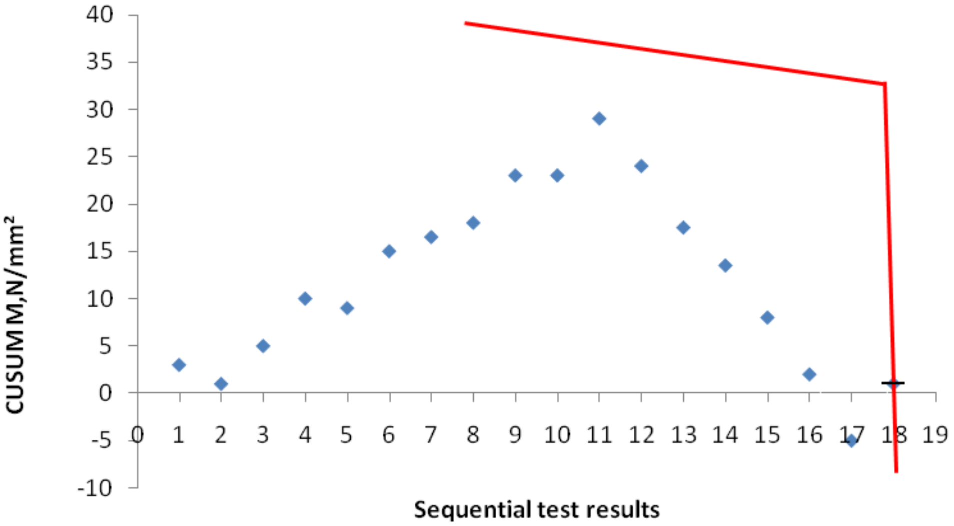
Example 5

A concrete has been specified as C25/30, maximum w/c ratio of 0.55, minimum cement content of 300 kg/m^3. Assuming a standard deviation of 4 N/mm^2 within a family of concretes a C25/30 concrete has a target strength of 38 N/mm^2 (cube). However the target w/c ratio of 0.53 equates to a target strength of 48 N/mm^2 and this higher value should be used as the target.

The measured strength is 44 N/mm^2. As there is an existing adverse trend in the results this result triggers action, Figure 12a). However if the target strength of this concrete had been incorrectly entered as 38 N/mm^2, instead of the result being 4 N/mm^2 below the target strength of 48 N/mm^2, it would appear 6 N/mm^2 above the target strength of 38 N/mm^2. This incorrect entry would not trigger action nor would it be identified as an outlier (current standard deviation is 4 N/mm^2 and the actual value is less than 3σ from the target value, see Figure 12b).



a) Correct strength entry for w/c controlled concrete



b) Incorrect strength entry for w/c controlled concrete

Figure 12: Example of the effects of incorrectly entering the target strength for a mix controlled by the maximum w/c ratio

For clarity, the transposition of these results to the equivalent value of the reference concrete has not been described.

8.4 Test rates

It is important that the appropriate level and frequency of testing is maintained in the control system. If the testing rate is too low, it will take too long to recognise that the system is no longer in control and there will be an unacceptable risk that non-conforming product is supplied to the customer. It would be uneconomic to test every single batch of concrete that is produced, so a test rate is required to produce a situation where the risk is shared reasonably between producer and user.

For a given rate of production, as the rate of testing increases the level of auto-correlation increases. Auto-correlation is the measure of the relationship between a result and previous results. A high level of auto-correlation indicates that a result is unlikely to be significantly different from the previous results (e.g. when two successive deliveries of the same type of concrete are sampled) and testing at a lower rate will provide the same level of control at a lower cost. However it has to be recognised that plants with low rates of production may not be able to achieve the optimal rate of testing. One way of handling low test rates is to have an additional cement differential, i.e. more cement than that indicated by the mix proportioning, to protect the user from the risk associated with low testing rates.

A test rate of about 16 results per month provides enough data to run an effective control system without an excessive level of auto-correlation.

The effect of auto-correlation is minimised in the calculation of the effect of a change in the control system by the use of an anti-hunting factor (see **5.5**).

8.5 Action following change

An investigation should be undertaken to determine the cause of the change. A CUSUM system will indicate the approximate time when the system started going out of control, aiding the investigation.

The over-riding principle is to protect the user of concrete. If the cause of an adverse change cannot be identified, all concretes in the family need to be adjusted. The appropriate increase or decrease in control cement content is established from the control system, the general rule is that upward changes in cement content are obligatory and downward changes are optional.

As explained in **5.5**, the appropriate action may be not to change the mix proportions, as the cause of the change has been already identified and corrected, e.g. the weigh scales have been repaired and re-calibrated. Good practice has been to have a minimum value of standard deviation, typically in the range 3.0 N/mm^2 to 3.5 N/mm^2. This means that when the control system shows the standard deviation is lower than this minimum value it is not adopted and the higher minimum value is assumed. With single concretes and short production runs, it is possible to achieve a standard deviation in the order of 2 N/mm^2. However, changes between batches of cement can result in changes of concrete strength in the order of 1 N/mm^2 and this equates of 0.5σ with a standard deviation of 2N/mm^2 and an increased risk of non-conformity. Having a minimum standard deviation protects the user and producer.

9 EN 206-1 Conformity Rules for Compressive Strength

9.1 Basic requirements for conformity of compressive strength

EN 206-1[3], the European standard for concrete, states that conformity assessment shall be made on test results taken during an assessment period that shall not exceed the last twelve months. Conformity of concrete compressive strength is assessed on specimens tested at 28 days[e] by applying two criteria:

Criterion 1 — applies to groups of n non-overlapping or overlapping consecutive test results f_{cm}[f] ;
Criterion 2 — applies to each individual test result f_{ci}[g] .

The criteria are given in Table 6.

Table 6: EN 206-1 Conformity Criteria

Production	Number n of test results for compressive strength in the group	Criterion 1	Criterion 2
		Mean of n results (f_{cm}) N/mm2	Any individual test result (f_{ci}) N/mm2
Initial	3	$\geq f_{ck} + 4$	$\geq f_{ck} - 4$
Continuous	Not less than 15	$\geq f_{ck} + 1.48\sigma$	$\geq f_{ck} - 4$

Depending upon the shape of the test specimens, the appropriate characteristic strength, f_{ck}, is selected for the specified compressive strength class. The same criteria apply regardless of whether cylinder or cube strengths are measured.

Based on criteria defined in EN 206-1, the production is defined as either ‘initial’ or ‘continuous’. Based on this classification of production, conformity is confirmed if both the criteria given in Table 6 are satisfied.

Where conformity is assessed on the basis of a concrete family (see **7.2**), Criterion 1 is to be applied to the reference concrete taking into account all transposed test results of the family; Criterion 2 is to be applied to the original test results, i.e. for conformity of the individual batch, the test result cannot be less than the characteristic strength associated with the specified compressive strength class minus 4 N/mm^2.

The UK Quarry Products Association (now part of the UK’s Mineral Products Association) has produced detailed Guidance on the application of the EN 206-1 conformity rules [4], and it is demonstrated that the risk to the concrete producer from the mean strength ($\geq$ (fck + 1.48σ)) criterion is unacceptably high for 15 results even when using a production margin of 1.96σ. Such a margin may be adequate for the individual batch criterion (equivalent to a 3 in a 1000 chance of failing when the standard deviation is 4.0 N/mm^2) but the

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[e] If the strength is specified for a different age the conformity is assessed on specimens tested at the specified age

[f] f_{cm} is the mean compressive strength of concrete

[g] f_{ci} is the individual test result for compressive strength of concrete

producer's risk of failing the mean strength criterion remains high. Simply increasing the rate of testing to some very high level does not solve the problem due to the increased level of auto-correlation off-setting the benefit from the increased number of test data.

9.2 Assessment period

Across Europe, the actual rate of testing in practice is variable. Test rates and lengths of conformity assessment periods have significant implications for conformity. At a rate of testing of 15 results a month, it would take about one month to detect that a change of 0.5σ lower than target had occurred, i.e. for this month the mean strength would have been, for example $(1.96 - 0.5)\sigma$, which would be a failure of the mean strength criterion if the assessment period was one month. By taking immediate action, the producer can adjust the mix proportions, e.g. increase the cement content, to give the expected target strength, and when the results are averaged over a longer period, the concrete will be in conformity for the whole of the assessment period. The fact that the same rules applied to the same data over different assessment periods can produce different conformity results, does not inspire confidence in the process.

The statistical basis of the mean strength criterion in EN 206-1 is the achievement of an AOQL of 5%. Failure to conform to the mean strength criteria shows that the average outgoing quality limit is more than 5% and not that the concrete is unfit for purpose. However it is an indication that the producer needs to take action to achieve an AOQL of 5%

9.3 Conformity rules for compressive strength

If the assessment period selected by the producer is one year and all the results in a family are used, the risk of non-conformity over the assessment period is close to zero. However over this year there could be periods where the production was not achieving the specified characteristic strength, see Figure 13.

Example 6

This example is based on conformity to the mean strength criterion of a C25/30 concrete with a target strength is 38 N/mm^2 (cube), i.e. a margin of 2σ. The standard deviation is a constant 4 N/mm^2. Figure 13 comprises randomly generated data with a mean strength of 38 N/mm^2 except for results 77 to 109 where the mean strength is set at 35 N/mm^2 for the number generation. However the data set was manually adjusted to exclude individual failures, as in practice such results are declared as non-conforming and excluded from the analysis of conformity of mean strength, i.e. they no longer form part of the population claimed to be in conformity .

The running mean of 15 indicates non-conformity between results 86 to 100 and this transposes to non-conformity of concrete represented by test results ((86 – 15) = 71 to 100) yet this period of 'non-conformity' is disguised when the assessment period is one year.

For the purposes of discussion, it is assumed that the data, and the concrete production, were not subject to production control using one of the control *chart systems described in this publication. If such a control system had* been in operation, *the change in mean strength would have been detected and the production would have been changed well before result 109.*

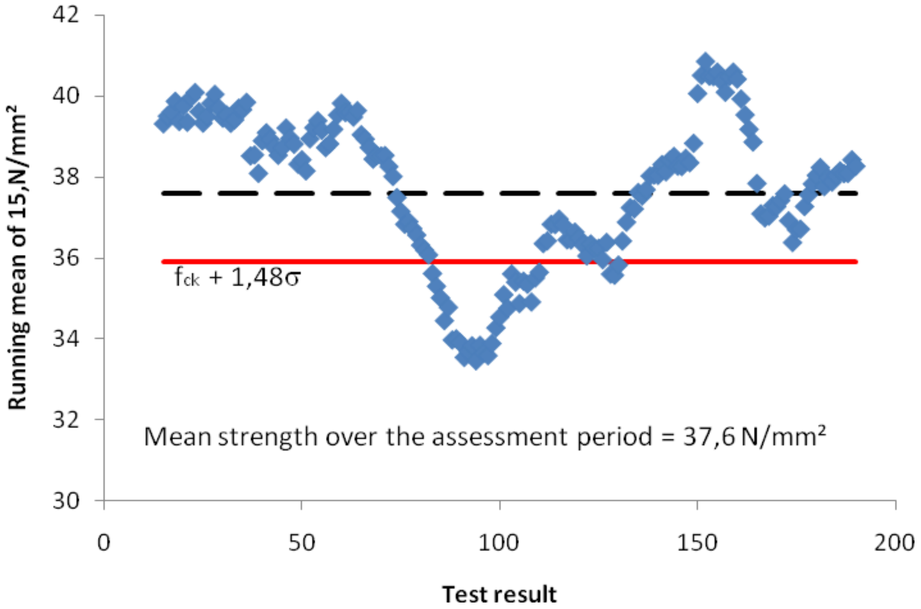

Figure 13: Illustration of the potential problem with using a long assessment period

Finding a solution that is fair to both consumers and producers is not easy as the solution has to cover normal, low and high production plants. A proposed solution is:

For plants with low rates of production of designed concrete
Where the number of test results per three months for designed concrete is not greater than 35, the assessment period shall comprise at least 15 and not more than 35 consecutive results taken over a period not exceeding 12 months.
For plants with high rates of production of designed concrete
Where the number of test results per three months for designed concrete exceeds 35, the assessment period shall not exceed three months.

Such a solution does not resolve all the issues related to conformity of strength, for example the criteria are based on the assumption that the concrete mix proportions are controlled by strength and this is not always the case (see **8.2**).

It is also proposed that the use of control charts be accepted as an alternative to the mean strength rule. This is conditional on the concrete being subject to third party certification or an agreement between the parties. As a control chart comprises successive sampling plans (with a known standard deviation), the operating-characteristic curve of the individual sampling plan may be established. The average outgoing quality (AOQ) curve is then determined by multiplying each percentage of all possible results below the required characteristic strength in the production by the corresponding acceptance probability.

Section **4.4** shows how this approach can be applied to control systems based on the Shewhart chart and **9.4** shows how it may be applied with the CUSUM system.

9.4 Achieving an AOQL of 5% with CUSUM

Caspeele and Taerwe [5] have developed a system where for a selected margin an upper limb V-mask for CUSUM M is used to determine when an AOQL of 5% is not being achieved. Parameters for the V-mask are given for *n* =15 and 35 and for independent and auto-correlated results. Production data show that concrete test results have some auto- correlation and therefore these are the appropriate values. For the reasons given in **9.2**, the producer should select an assessment period based on 35 results. The paper [5] is focussed on achieving an AOQL of 5% and it includes margins less than 1.64σ. The use of a margin less than 1.64σ means that the producer is deliberately targeting the production not to achieve the specified characteristic strength. This is unacceptable and the margin should never be less than 1.64σ. Table 7 gives the V-mask parameters for selected margins based on *n* =35 and auto-correlated results.

Table 7: Selected conformity V-mask parameters

Margin	Decision interval	Slope
1.64σ	3σ	σ/2
1.66σ	8σ	σ/4
1.70σ	9σ	σ/4
1.71σ	4σ	σ/2
1.74σ	10σ	σ/4
1.76σ	5σ	σ/2
1.82σ	6σ	σ/2
1.86σ	7σ	σ/2
1.91σ	8σ	σ/2
1.95σ	9σ	σ/2
1.99σ	10σ	σ/2
2.06σ	2σ	σ/1
2.26σ	5σ	σ/1
2.49σ	10σ	σ/1

The conformity V-mask only applies to the upper limb, i.e. to the actual strength being less than the target strength, and its length is the selected value of *n*; 35 in the case of Table 7. In effect, the V-mask tests whether the last *n* results have achieved an AOQL of 5%. Figure 14 for CUSUM M shows the conformity V-mask with the V-mask that shows when a significant change has happened (see **5.4**).

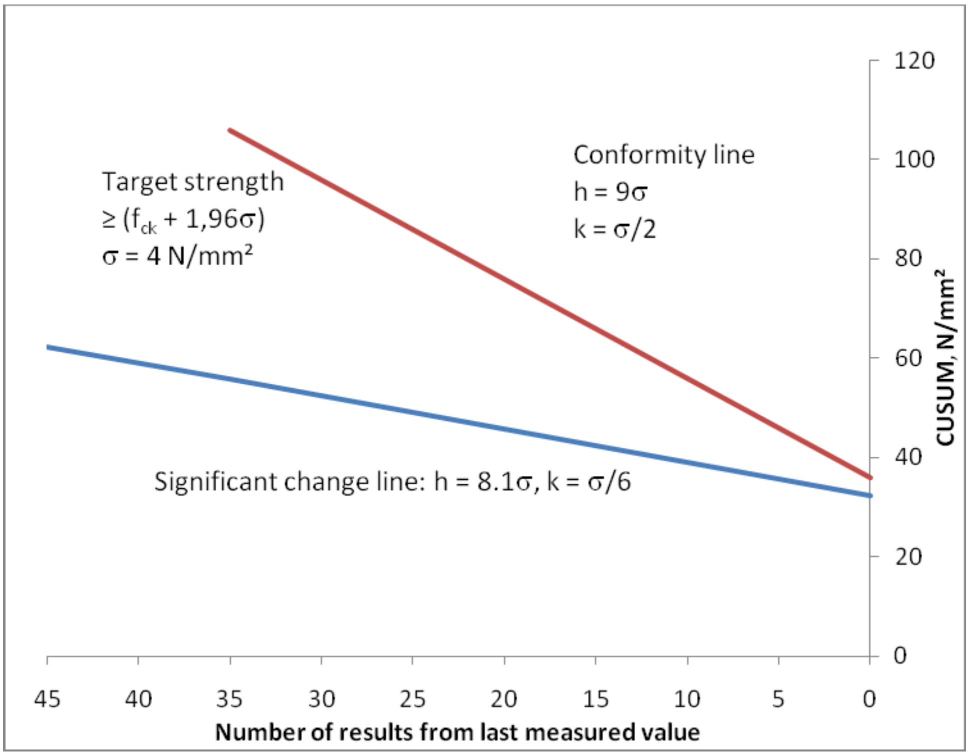

Figure 14: Conformity and action V-masks for a target strength of (f_{ck} + 1,96σ)

When the CUSUM crosses the significant change line, the producer takes immediate appropriate action. However there will be a number of test specimens that have been cast but not tested. If a conformity V-mask is being used, the CUSUM M should be re-set once these results have been tested. This means that the adverse trend may move towards the conformity line. At a test rate of 16 per month and the use of 7-day strength data, there would be about 4 results between crossing the significant change line and re-setting CUSUM M and therefore the probability of crossing the conformity line is very low.

A decade of experience using CUSUM with the target strength and significant change limits given in Figure 14 and checking conformity of mean strength to EN 206-1:2000 using a running mean of 35 results [19] has never resulted in non-conformity of mean strength. Under these conditions the conformity V-mask is highly unlikely to be crossed and if it was crossed, it would indicate that the producer must take action to achieve the target strength, but the producer would have taken this action already, i.e. when the significant change line was crossed.

9.5 Non-conformity

If the control system shows that an AOQL of 5% is not being achieved, the producer is required to take immediate action to achieve the target strength. In addition the producer should identify any concretes that are not fit for purpose and inform the user and specifier. The certification body will check that the investigation was undertaken in the appropriate way and the users and specifiers correctly informed.

10 Implementing Control Systems

Once continuous production is established the producer should have the choice of controlling the mean strength using control charts. The use of control charts should be limited to producers with third party certification (e.g. the majority of the ready-mixed concrete and precast concrete industries in Western Europe) or to where there is agreement between the producer and user.

The following are recommended minimum requirements on the control system:

- achieve a maximum average outgoing quality (AOQ) not exceeding 5,0%; (*Comment: This ensures that no more than 5% of the production is below the specified characteristic strength).*
- aim to ensure conformity of the production with the required characteristic strength;
- include regular monitoring of strength and standard deviation and deviations from target values;
- where applicable, include one or more procedures for speeding the response of the system (e.g. use of early strength data, use of concrete families);
- define and apply clear decision rules for conformity and warning limits;
- document how the system achieves a maximum average outgoing quality (AOQ) not exceeding 5.0% (unless one of the rules of application given in an informative annex to EN 206-1 is used);
- when the control chart shows that the standard deviation is ≥ 0.5 N/mm^2 above the currently applied value, change the applied value.

11 CUSUM Example

11.1 Reference mix and concrete family

The following example is used to simply illustrate the CUSUM process applied to a concrete plant controlling its production based on a family of mixes, as is the common practice in the UK.

The control system is based on a reference concrete described in Table 8. This concrete is representative of the main concrete produced at the plant. The control cement content is the current level that the CUSUM identifies as necessary to produce the target strength of the reference concrete. Note that concrete with the same compressive strength class as the reference concrete might be actually produced with higher cement contents than the reference concrete. For example, specification requirements for durability may show that the minimum cement content or the cement content to satisfy the specified maximum w/c ratio is higher than the CUSUM control cement content of the reference concrete. How these mixes are handled is described in **8.3**.

Table 8: Reference Mix Details

Compressive strength	C32/40
Aggregate size and type	20mm gravel
Cement Type	CEM III/A
Slump	70mm (S2)
Admixture	None
Control cement content	320 kg/m^3

The control cement content is that expected to give the target strength of the reference concrete. Not all concretes will be included in the family of mixes used to control the main production. The parameters of the family are shown in Table 9.

Table 9: Family parameters

Compressive strength	C16/20 to C45/55 inclusive
Aggregate size and type	Gravel only 20mm or 10mm
Cement Type	CEM III/A only
Slump	25mm to 150mm inclusive
Admixture	With or without water reducing admixture

11.2 Main relationship

A key relationship needed in the CUSUM analysis is the main relationship between cement content and strength (Figure 15), which not only allows mixes to be transposed to an equivalent to the reference concrete, but is also used to determine the size of correction to be applied when the CUSUM indicates change has occurred (see **5.6**).

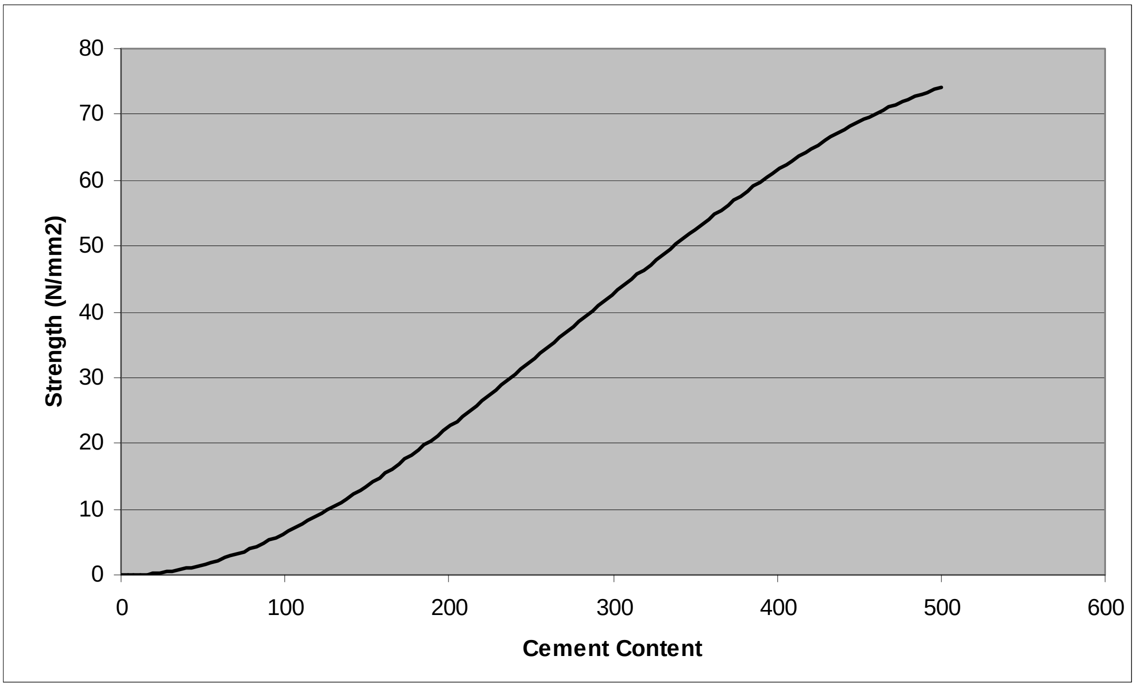

Figure 15: Main relationship between cement content and strength

In addition to the main relationship, relationships are also needed to be established for the effect on cement content of changing the aggregate size, slump and the effect of the water-reducing admixture (WRA). These effects are expressed as adjustments and are made to the cement content. The magnitude of these adjustments will normally be established by laboratory work. The values for the family of mixes in this example are shown in Table 10. The adjustments are those needed to convert the cement content of the tested concrete into an equivalent cement content on the main strength/cement content relationship (Figure 15). For example a 10mm maximum aggregate size concrete would have a higher cement content than a 20mm maximum aggregate size concrete (the aggregate size used in the main relationship) and so a 10mm concrete has to be adjusted by reducing the cement content used to produce that concrete to what it would have been if a 20mm concrete had been produced. If a WRA is used, the cement content is adjusted upwards. In this example the WRA is added as a constant percentage by mass of cement.

Once these changes have been made to adjust the actual cement content of the tested concrete to the equivalent value on the main relationship, a second adjustment is needed to further correct that new cement content to that of the reference concrete. This strength adjustment is the difference in strength on the main relationship between that at the equivalent cement content of the tested concrete and the target mean strength of the reference concrete.

<table>
<tr><th colspan="7">Table 10: Adjustments per cubic metre to convert the cement content of the tested concrete to a concrete on the main relationship[A]</th></tr>
<tr><td colspan="7">Adjustments where the tested concrete contained a WRA</td></tr>
<tr><td colspan="2">Cement content of tested concrete, kg/m^3</td><td colspan="2">200 to 380</td><td colspan="3">380+</td></tr>
<tr><td colspan="2">Adjustment to cement content, kg/m^3</td><td colspan="2">+25 kg</td><td colspan="3">At these higher cement contents it is more effective to use a superplasticizing admixture[B].</td></tr>
<tr><td colspan="7">Adjustments where the tested concrete contained 10mm maximum aggregate size[C].</td></tr>
<tr><td colspan="2">Cement content of tested concrete, kg/m^3</td><td colspan="2">200 to 380</td><td colspan="3">380+</td></tr>
<tr><td colspan="2">Adjustment to cement content, kg/m^3</td><td colspan="2">−15</td><td colspan="3">−10</td></tr>
<tr><td colspan="7">Adjustments where the tested concrete has a target slump that is not 70mm</td></tr>
<tr><td>Target slump, mm</td><td>20 (S1)</td><td>50</td><td>70 (S2)</td><td>100</td><td>120 (S3)</td><td>150</td></tr>
<tr><td>Slump adjustment</td><td>+15 kg</td><td>+10 kg</td><td>0</td><td>−5 kg</td><td>−10 kg</td><td>−15 kg</td></tr>
<tr><td colspan="7">[A] Numerically equal but opposite in sign (+ becomes -) adjustments are used in mix proportioning.
[B] As the following example does not need this adjustment for superplasticizer, a value is not given.
[C] As the following example does not include aggregates larger than 20mm, no adjustments are given.</td></tr>
</table>

11.3 Applying adjustments

Consider the following concretes tested for inclusion in the CUSUM system

Mix Ref 1

C25/30 20mm aggregate CEM III/A 100 mm slump no WRA

The mix was produced with a cement content of 275 kg/m^3. This mix meets the family criteria in Table 9.

To relate the mix to the reference concrete described in Table 8, the mix must first be adjusted for the fact it was specified as 100mm slump rather than 70mm slump. Table 10 indicates that for 100mm slump an adjustment of −5 kg/m^3 is required. The adjustment is negative as additional cement would have been added to the mix to maintain the w/c ratio after the consistence had been increased from 70mm to 100mm slump.

The adjusted cement content becomes: 275 − 5 = 270 kg/m^3

From the main relationship in Figure 15, we would expect a 270kg/m^3 cement content to achieve a strength of 37.3[h]N/mm^2 but the reference concrete is

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[h] This is a value calculated from the equation of the line in Figure 15.

C32/40. As the plant has a standard deviation of 3.5N/mm^2 and a design margin of 2.0σ, the target mean strength of the reference concrete is 47 N/mm^2. A difference of 9.7 N/mm^2 (47 – 37.3 N/mm^2) in the CUSUM system has to be introduced to transpose the strength at the equivalent cement content to the target mean strength of the reference mix. For example in Table 11 mix reference 1, the predicted and actual 28-day strengths are 42.5 N/mm^2 and 39.5 N/mm^2 respectively. After adjustment to equivalent cement content values of the reference concrete, these become 52.2 N/mm^2 (42.5 + 9.7) and 49.2 N/mm^2 and the change in CUSUM M is 52.2 – 47 = 5.2 N/mm^2 if the predicted strength is being used and 49.2 – 47 = 2.2 N/mm^2 when the predicted 28-day strength is replaced with the actual 28-day strength.

In this example the lowest compressive strength class in the concrete family is C16/20. However, if a family were to include concrete with strength classes lower than C16/20, i.e. a target mean strength of less than 27 N/mm^2, any results are re-calculated using the standard deviation obtained from Figure 3.

Mix Ref 2

C32/40 20mm aggregate CEM III/A 150mm slump with a WRA

The mix was produced with a cement content of 310kg/m^3. This mix meets the family criteria in Table 9.

To relate the mix to the reference concrete described in Table 8, the mix must first be adjusted for the fact that it contained a WRA; and then adjusted because it was specified as 150mm slump rather than 70mm slump. For the WRA the cement content for CUSUM purposes will need to be increased to correct for the water reduction effected by the addition of the WRA. For the higher-than-reference slump the adjustment in cement content is negative.

Total adjustment to apply is +25 –15 = 10kg (from Table 10)

The adjusted cement content becomes:
310 + 10 = 320 kg/m^3

Note when undertaking mix proportioning, the adjustments from the main relationship always have the opposite sign, but the same numerical value.

The second adjustment is to correct the recorded strength at this increased cement content to the strength expected at the cement content of the reference mix. From the main relationship in Figure 15, a 320 kg/m^3 cement content is expected to achieve a strength of 46.8^{h}N/mm^2 but the reference concrete is C32/40. As the plant has a current standard deviation of 3.5N/mm^2 and a design margin of 2.0σ, the target mean strength of the reference concrete is 47 N/mm^2, a difference of 0.2 N/mm^2, which is the adjustment that will be made to the predicted and actual cube strengths in the CUSUM system.

Mix Ref 3

C32/40 20mm aggregate CEM III/A 70mm slump no WRA

The mix was produced with a cement content of 320 kg/m^3. This mix is the reference concrete and as it was batched at the control cement content, no adjustments need to be applied.

These concretes are the first three included in the CUSUM analysis tabulated in Table 11.

11.4 CUSUM calculation

Once the adjustments have been made and the adjusted 28 day strength calculated, the data may be used in any control system (e.g. CUSUM or Shewhart). The following example analyses the data by CUSUM techniques. A CUSUM is run on mean (CUSUM M), standard deviation (CUSUM R) and correlation (CUSUM C). For control purposes the mixes include a prescribed concrete (P300, sample reference 13) and a nominal mix (1:2:4, sample reference 14).

The results are plotted on the CUSUM and the V-mask overlaid on each result. This is a manual system used to illustrate the principles of the CUSUM technique. Clearly, this process would normally be carried out on computer, either through a spreadsheet, a commercially available CUSUM programme or by the development of a company specific computer programme.

The current plant standard deviation is 3.5 N/mm^2 which gives a target range of (1.128 x 3.5) = 3.9 N/mm^2 (see **2.3**). The margin is 2.0σ. The target cube strength of the reference concrete is (40 + (2.0 x 3.5)) = 47 N/mm^2.

From the three plots in Figure 16, Figure 17 and Figure 18 it may be seen that the correlation and standard deviation are running in control, but there has been a change in the mean from point 7 to point 17. This is evidence that the current control cement content of 320 kg/m^3 is not giving the required average strength of 47 N/mm^2. As it is the upper arm of the V-mask that has been crossed, this indicates that the average mean strength is lower than the target strength. The mix proportions are immediately adjusted (see **11.5**) and CUSUM M is reset at zero. Note that this change has been made on the basis of a set of actual 28 day strengths (data points 1 to16) and predicted 28 day strengths (data point 17).

Table 11: CUSUM calculation

Mix description (all CEM III/A)						Results			Adjustments									CUSUM M			CUSUM R				CUSUM C	
Mix reference	**Strength class**	**Aggregate size**	**Target slump**	**Plasticiser**	**Batched cement content**	**Actual 7 day**	**Predicted 28 day**	**Actual 28 day**	**Total cement adjustment**	**Adjusted cement content**	**Cement/Strength Code**	**Expected strength**	**Reference strength**	**Target strength**	**Strength adjustment**	**From predicted**	**From actual**	**Adjusted strength**	**Difference from target**	**CUSUM M**	**Range**	**Target Range**	**Difference from target**	**CUSUM R**	**Actual — predicted 28 day**	**CUSUM C**
CURRENT STANDARD DEVIATION = 3,5 N/mm^2; TARGET RANGE = 1,128 x 3,5 = 3,9 N/mm^2																										
1	C25/30	20	100	No	275	31.1	42.5	39.5	-5	270	A	37.3	40	47	9.7		49.2	49.2	2.2	2.2					-3.0	-3.0
2	C32/40	20	150	Yes	310	33.8	45.3	46.3	10	320	A	46.8	40	47	0.2		46.5	46.5	-0.5	1.7	2.7	3.9	-1.2	-1.2	1.0	-2.0
3	C32/40	20	70	No	320	35.2	46.8	46.8	0	320	A	46.8	40	47	0.2		47.0	47.0	0.0	1.7	0.5	3.9	-3.4	-4.6	0.0	-2.0
4	C32/40	20	70	No	320	37.2	48.8	49.3	0	320	A	46.8	40	47	0.2		49.5	49.5	2.5	4.2	2.5	3.9	-1.4	-6.0	0.5	-1.5
5	C25/30	20	70	Yes	245	26.7	37.5	39.5	25	270	A	37.3	40	47	9.7		49.2	49.2	2.2	6.4	0.3	3.9	-3.6	-9.6	2.0	0.5
6	C32/40	20	150	Yes	310	41.5	52.8	53.8	10	320	A	46.8	40	47	0.2		54.0	54.0	7.0	13.4	4.8	3.9	0.9	-8.7	1.0	1.5
7	C32/40	20	70	No	320	42.6	53.8	53.3	0	320	A	46.8	40	47	0.2		53.5	53.5	6.5	19.9	0.5	3.9	-3.4	-12.1	-0.5	1.0
8	C28/35	20	50	No	285	28.2	39.2	39.2	10	295	A	42.1	40	47	4.9		44.1	44.1	-2.9	17.0	9.4	3.9	5.5	-6.6	0.0	1.0
9	C28/35	20	50	No	285	30.9	42.2	40.7	10	295	A	42.1	40	47	4.9		45.6	45.6	-1.4	15.6	1.5	3.9	-2.4	-9.0	-1.5	-0.5
10	C40/50	20	120	Yes	360	40.4	51.8	48.8	15	375	A	57.3	40	47	-10.3		38.5	38.5	-8.5	7.1	7.1	3.9	3.2	-5.8	-3.0	-3.5
11	C25/30	20	100	No	275	27.6	38.6	40.5	-5	270	A	37.3	40	47	9.7		50.2	50.2	3.2	10.3	11.7	3.9	7.8	2.0	1.9	-1.6
12	C25/30	20	70	Yes	245	24.1	34.5	35.0	25	270	A	37.3	40	47	9.7		44.7	44.7	-2.3	8.0	5.5	3.9	1.6	3.6	0.5	-1.1
13	P300	20	150	Yes	300	26.2	36.9	37.4	10	310	A	44.9	40	47	2.1		39.5	39.5	-7.5	0.5	5.2	3.9	1.3	4.9	0.5	-0.6
14	1:2:4	20	70	No	270	27.6	38.6	37.6	0	270	A	37.3	40	47	9.7		47.3	47.3	0.3	0.8	7.8	3.9	3.9	8.8	-1.0	-1.6
15	C40/50	20	120	Yes	360	38.3	49.8	47.3	15	375	A	57.3	40	47	-10.3		37.0	37.0	-10.0	-9.2	10.3	3.9	6.4	15.2	-2.5	-4.1
16	C40/50	20	120	Yes	360	41.5	52.8	53.8	15	375	A	57.3	40	47	-10.3		43.5	43.5	-3.5	-12.7	6.5	3.9	2.6	17.8	1.0	-3.1
17	C25/30	20	100	No	275	21.7	31.5		-5	270	A	37.3	40	47	9.7	41.2		41.2	-5.8	-18.5	2.3	3.9	-1.6	16.2		
TARGET STRENGTH NOT BEING ACHIEVED; CEMENT CONTENT INCREASED; CUSUM M RESET TO ZERO																										

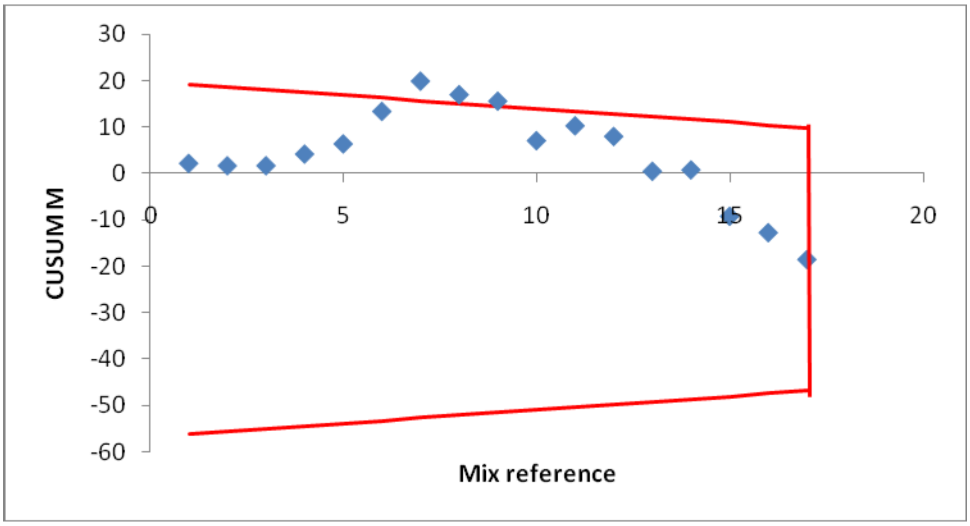

Figure 16: CUSUM M

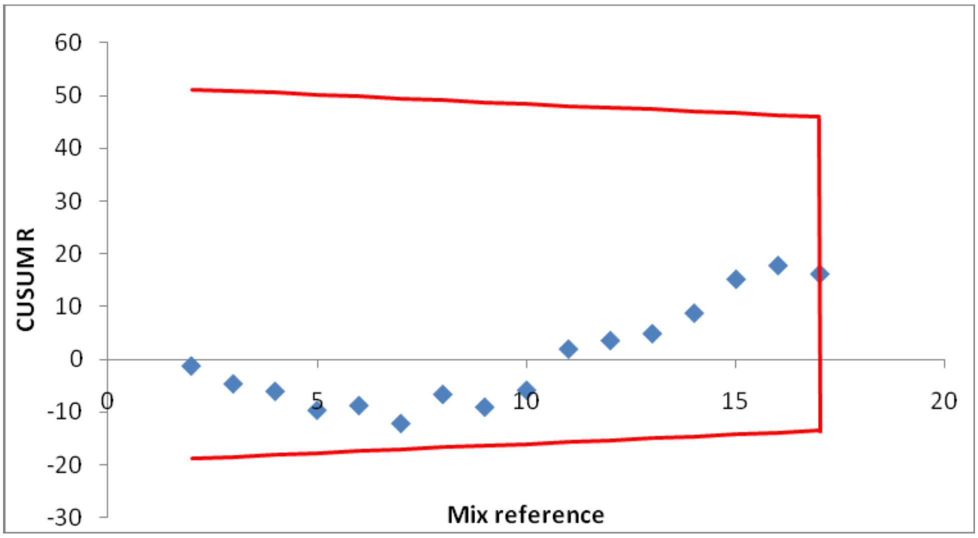

Figure 17: CUSUM R

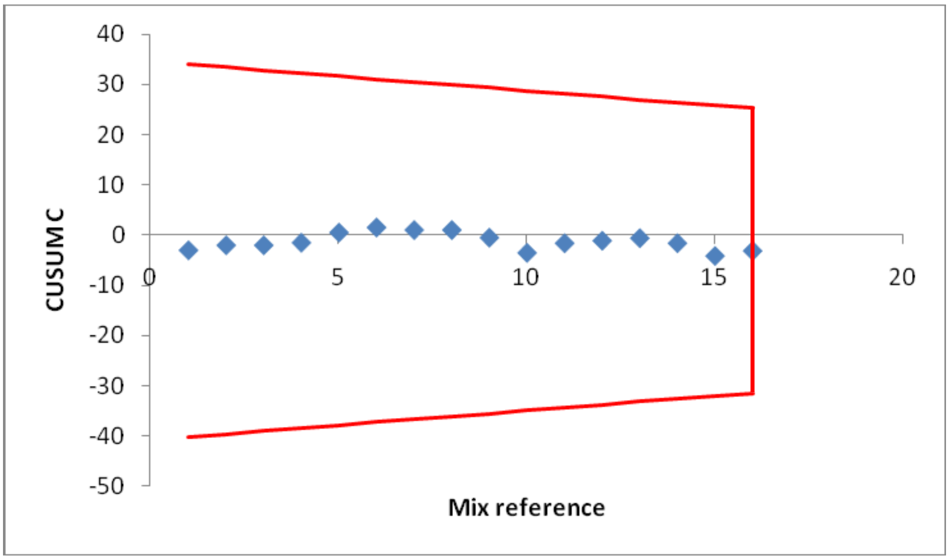

Figure 18: CUSUM C

11.5 CUSUM action following change

The CUSUM M has shown that there has been a decline in the performance; therefore to bring the process back into control it is necessary to increase the control cement content. The magnitude of the increase in cement content is a function of the standard deviation of the plant, and the number of results over which the change has taken place (see **5.6**).

In this case, the plant standard deviation is 3,5 N/mm^2 and the change occurred at Mix reference 7, but the CUSUM M first crosses the V-mask at Mix reference 9 giving a change over 9 results. From Figure 19 it can be seen that a change over 9 results gives a change in cement content of 14 kg/m^3. For simplicity, this would be rounded to 15 kg/m^3 and therefore the control cement content of the reference concrete would be increased from 320 kg/m^3 to 335 kg/m^3.

A new main relation would also be adopted that relates to a control cement content of 335 kg/m^3 for a characteristic strength of 40 N/mm^2 (target strength 47 N/mm^2). Table 12 shows the relationships in tabular form and from Table 13, this can be seen as a change in cement/strength code from A to B. The cement contents actually used at the plant would immediately be increased to the amount shown by the new main relationship. This changed main relationship will also lead to revised adjustments being applied to obtain the predicted cube strength of the reference concrete. These adjustments are applied from result 18 onwards (Table 13). They will also be applied to the batching of new mixes, but there will be a period where the concrete has already been batched at a cement content that is less than that now known to be necessary. However for the control of the mean strength, mix 18 onwards is adjusted to the new target cement content of the reference concrete (335 kg/m^3).

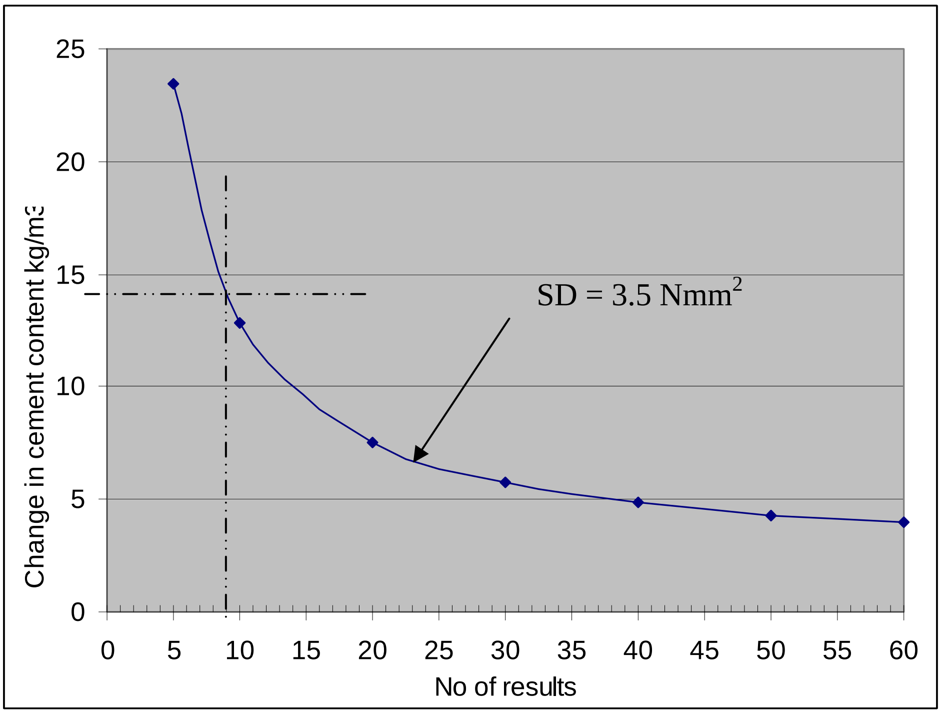

Figure 19: Strength change against number of results

Table 12: Relationships between strength and cement content

Cube strength N/mm^2	Cement content, kg/m^3, for cement/strength codes		Cube strength N/mm^2	Cement content, kg/m^3, for cement/strength codes	
	A	B		A	B
20	180.0	195.0	41	290.0	305.0
21	185.0	200.0	42	295.0	310.0
22	190.0	205.0	43	300.0	315.0
23	195.0	210.0	44	305.0	320.0
24	200.0	215.0	45	310.0	325.0
25	205.0	220.0	46	315.0	330.0
26	210.0	225.0	47	320.0	335.0
27	215.0	230.0	48	325.0	340.0
28	220.0	235.0	49	330.0	345.0
29	225.0	240.0	50	335.0	355.0
30	230.0	245.0	51	340.0	360.0
31	235.0	255.0	52	345.0	365.0
32	240.0	260.0	53	355.0	370.0
33	245.0	265.0	54	360.0	375.0
34	255.0	270.0	55	365.0	380.0
35	260.0	275.0	56	370.0	385.0
36	265.0	280.0	57	375.0	390.0
37	270.0	285.0	58	380.0	395.0
38	275.0	290.0	59	385.0	400.0
39	280.0	295.0	60	390.0	405.0
40	285.0	300.0			

11.6 Further data and a change in standard deviation

Table 13 is the continuation of the CUSUM calculation with additional data. During this period no more actual 28-day strength are available. The CUSUMs with the additional data are shown in Figures 20 to 22.

Following the change in cement content to achieve the target strength, sample number 18, C32/40 70mm slump, which is the control mix and therefore batched at the control cement content has previously not required a correction to the strength (See Table 11, samples 3, 4 and 7). However, the control cement content has now increased to 335 kg/m^3; since the mix was batched at 320 kg/m^3 before the CUSUM M detected the need for a change, an adjustment from the new main relation needs to be applied.

The same concrete was batched at sample number 22 but in this case the control mix cement content changes needed to compensate for the changes in mean strength and in standard deviation have already been implemented. For this reason there is no adjustment to make as the batched cement content is now 340 kg/m^3 (320 +15 +5)(+15 kg/m^3 necessitated by the change in strength/cement content relationship and 5 kg/m^3 necessitated by the increase in standard deviation, see Table 12).

The range over the adjustment between samples 17 and 18 in compressive strength is large (56.3 — 41.2) = 15.1 N/mm^2. The results immediately before and after the change of mean strength are corrected on different main relationships which will increase variability. Consequently an amendment needs to be made to the result immediately prior to the change mean strength in order to not introduce excessive variation into the CUSUM R calculation. The result before the change of mean strength is adjusted using the new main relation for the range calculation only. From the new main relationship the expected strength is 44.2 N/mm^2 and this reduces the range from 15.1 N/mm^2 to 12.1 N/mm^2.

After Mix reference 18, a change in the plant standard deviation is also detected, see Figure 21. A new standard deviation needs to be calculated from the average current range. The average range is 5.3 N/mm^2 and the new standard deviation is 4.7 N/mm^2 (5.3/1.128). To avoid over-correcting, a decision is taken to change the standard deviation to 4.0 N/mm^2. The margin is increased to 1.96 x 4.0 = 7.8 N/mm^2 rounded to 8 N/mm^2. A 1 N/mm^2 increase in the margin requires a 5 kg/m^3 increase in the cement content, see Table 12. The current control mix cement content is therefore immediately increased from 335 kg/m^3 to 340 kg/m^3. The cement/strength relationship is unchanged (relationship B); what has changed is the target strength of the reference concrete has moved from 47 to 48 N/mm^2.

Table13: CUSUM table continued

Mix description (all CEM III/A)						Results			Adjustments									CUSUM M			CUSUM R				CUSUM C	
Mix reference	Strength class	Aggregate size	Target slump	Plasticiser	Batched cement content	Actual 7 day	Predicted 28 day	Actual 28 day	Total cement adjustment	Adjusted cement content	Cement/Strength Code	Expected strength	Reference strength	Target strength	Strength adjustment	From predicted	From actual	Adjusted strength	Difference from target	CUSUM M	Range	Target Range	Difference from target	CUSUM R	Actual – predicted 28 day	CUSUM C
16	C40/50	20	120	Yes	360	41.5	52.8	53.8	15	375	A	57.3	40	47	-10.3		43.5	43.5	-3.5	-12.7	6.5	3.9	2.6	17.8	1.0	-3.1
17	C25/30	20	100	No	275	21.7	31.5		-5	270	A	37.3	40	47	9.7	41.2		41.2	-5.8	-18.5	2.3	3.9	-1.6	16.2		
TARGET STRENGTH NOT BEING ACHIEVED; CEMENT CONTENT INCREASED; CUSUM M RESET TO ZERO																										
17	Adjusted					21.7	31.5			270	B	34.3	40	47	12.7	44.2		44.2		0.0				16.2		
18	C32/40	20	70	No	320	41.8	53.1		0	320	B	43.8	40	47	3.2	56.3		56.3	9.3	9.3	12.1	3.9	8.2	24.4		
STANDARD DEVIATION INCREASED TO 4,0 N/mm^2; TARGET STRENGTH INCREASED TO 48 N/mm^2; TARGET RANGE INCREASED TO 4,5 N/mm^2; CEMENT CONTENT INCREASED																										
18	Adjusted					41.8	53.1			320	B	43.8	40	48	4.2	57.3		57.3		9.3				0.0		
19	C25/30	20	100	No	290	26.2	36.9		-5	285	B	37.2	40	48	10.8	47.7		47.7	-0.3	9.0	9.6	4.5	5.1	5.1		
20	C28/35	20	50	No	305	28.6	39.7		10	315	B	42.9	40	48	5.1	44.8		44.8	-3.2	5.8	2.9	4.5	-1.6	3.5		
21	P300	20	150	Yes	300	24.4	34.8		10	310	B	41.9	40	48	6.1	40.9		40.9	-7.1	-1.3	3.9	4.5	-0.6	2.9		
22	C32/40	20	70	No	340	39.5	51.0		0	340	B	47.6	40	48	0.4	51.4		51.4	3.4	2.1	10.5	4.5	6.0	8.9		

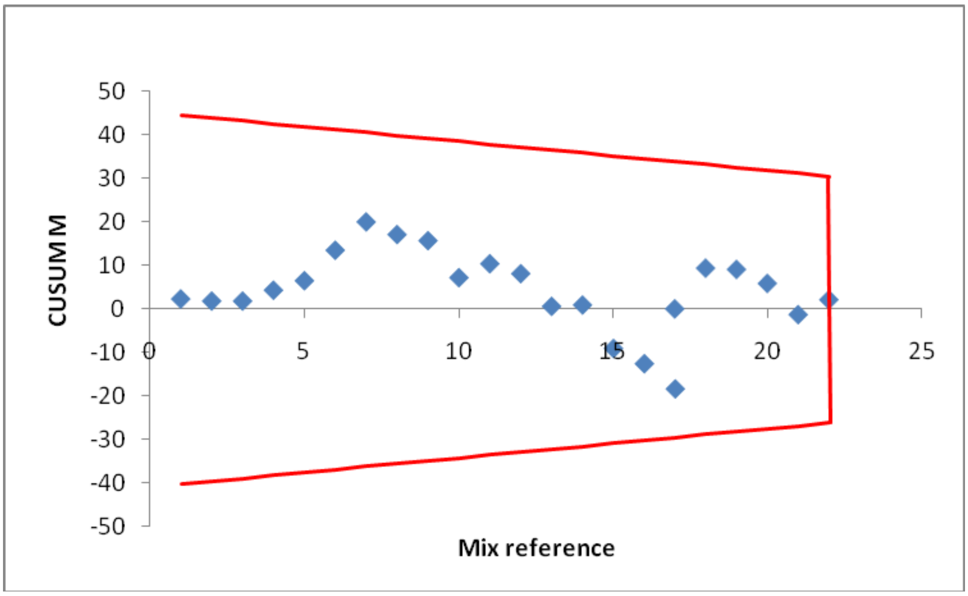

Figure 20: CUSUM M with additional data

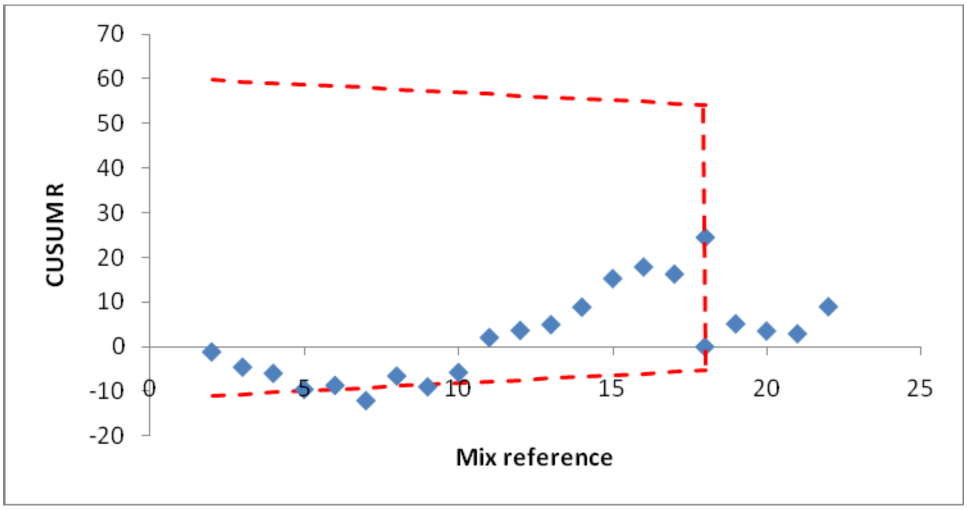

Figure 21: CUSUM R with additional data

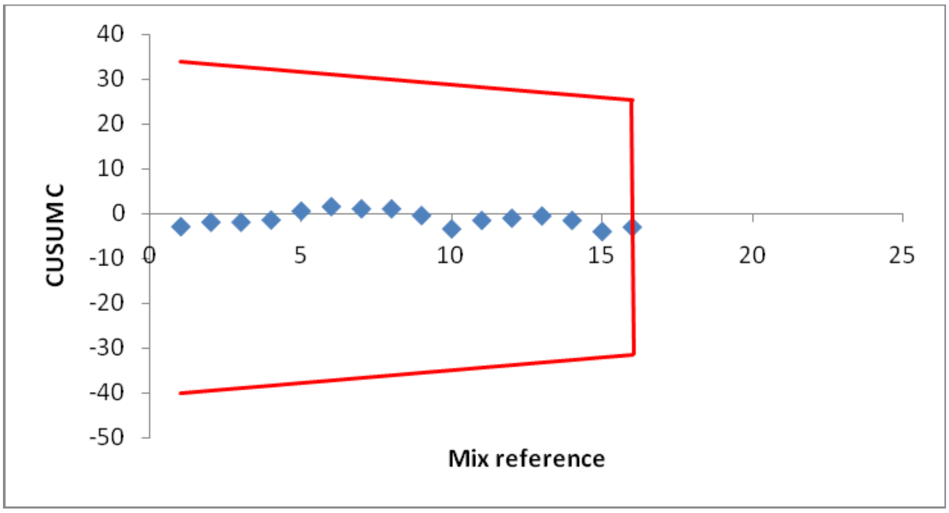

Figure 22: CUSUM C with additional data

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